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ABSTRACT

The Minnesota School Mathematics and Science Teaching (MINNEMAST) Project is characterized by its emphasis on the coordination of mathematics and science in the elementary school curriculum. Units are planned to provide children with activities in which they learn various concepts from both subject areas. Each subject is used to support and reinforce the other where appropriate, with common techniques and concepts being sought and exploited. Content is presented in story fashion. The stories serve to introduce concepts and lead to activities. Imbedded in the pictures that accompany the stories are examples of the concepts presented. This unit introduces fractions by comparing different scales on number lines. The concept is built as a ratio between scales. As in the earlier units, operations with fractions are presented through activities involving the number line; the four operations, addition, subtraction, multiplication and division of fractions, are covered in this unit. Worksheets and commentaries to the teacher are provided and additional activities are suggested. (JP)

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Math And Science Teaching project

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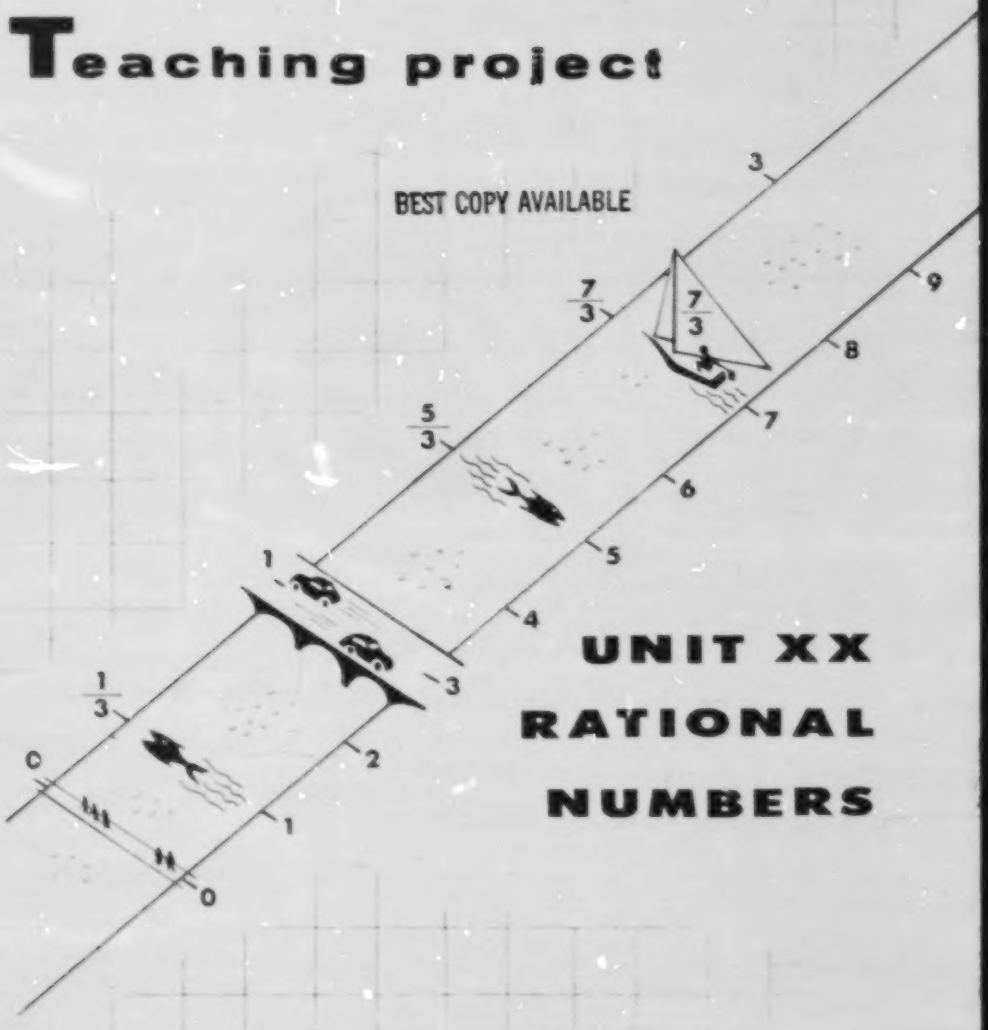

UNIT XX
RATIONAL NUMBERS

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1965

SE 017 537

MATHEMATICS
FOR THE
ELEMENTARY SCHOOL

UNIT XX

Rational Numbers

The Minnesota School Mathematics and Science Teaching Project

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MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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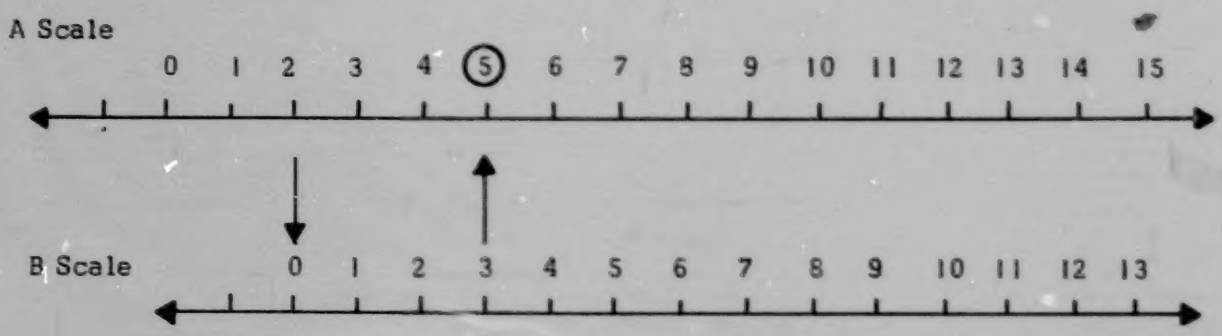
Irrational Numbers

Teacher's Introduction

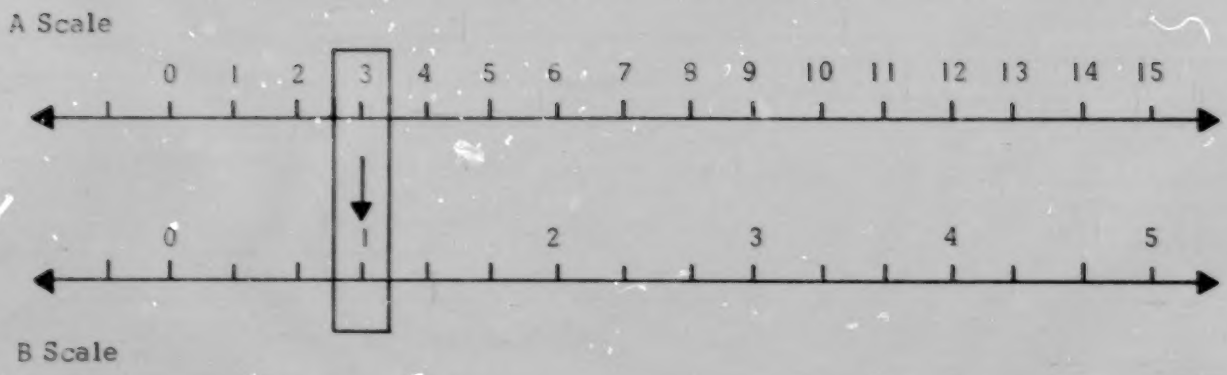
Unit XX is based upon the concept of the number line which has been developed in previous Minnemast units. This concept has provided a link between geometric and algebraic methods of addition and subtraction. For example, in order to find the sum of 2 + 3, we use two number scales:



Locate point 2 on scale A. Slide the bottom scale (you could just as well do this with the top one) until the point marked "0" corresponds to that point 2 on the top scale.

Locate point "3" on the bottom scale. Then read the number "5" on the top scale. Thus, 2 + 3 = 5. Actually this method defines addition of two numbers even when they are not integers. This will be demonstrated later.

In this unit we introduce multiplication of rational numbers by use of the number line. For example, suppose we wanted to multiply 2 · 3. We would construct the following scales:



These scales show that point "1" of "B" scale corresponds to point "3" of "A" scale (1⟶3). Notice that "zero" of "B" scale is directly below "zero" of "A" scale.

Going from a point on B scale to a corresponding point on A scale is multiplication by 3. Therefore, to find the product of $2 \cdot 3$, locate point 2 on B scale, then read its coordinate on A scale. This will give you the point 6, the product of 2 and 3.

It is also possible to look at these same scales in another way. We see that 3 on the top scale goes to 1 on the bottom scale; this indicates division by 3. If we want to divide a number b by 3, all we have to do is to locate b in the top scale, then look directly under it for $\frac{b}{3}$. For example, to find $\frac{6}{3}$ we locate 6 in the top scale, then look directly under it we see the number 2, therefore, $\frac{6}{3} = 2$.

In general, we can say that if we have two scales which have the same "0" point and point "1" in one scale corresponds to a number a in the second scale, then going from the first to the second scale is defined as multiplication by the number a. Conversely, going from the second scale to the first scale is defined as division by a. The key to this unit is this: If 1⟶a, this defines multiplication by a; if a⟶1, this defines division by a (provided that $a \neq 0$). The advantage of this method is that it enables you to divide

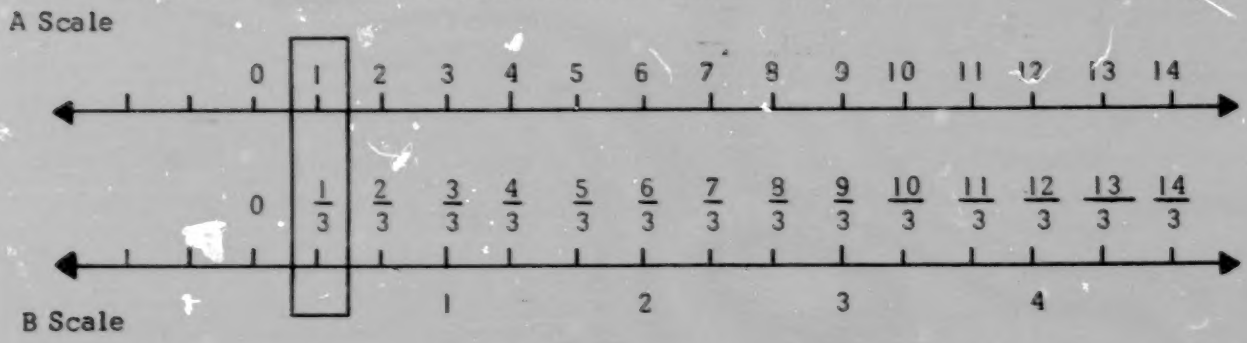
and multiply numbers other than the integers.

Now let us consider these other numbers. We have said that the key to this scale is $3 \longrightarrow 1$, which means that when we go from a point on A scale to a corresponding point on B scale we are dividing by 3. We can now name the point on B scale which is directly under "1" on A scale, "$\frac{1}{3}$". We have named it according to the operation we have performed: one divided by three.

Similarly, we can give names to the other points on the B scale. Any number that you can construct by the above process is called a rational number. Note that all these numbers have the form $\frac{a}{b}$, (where $b \neq 0$).

Our definitions of addition and multiplication are now applicable to these new numbers, the rational numbers. As an illustration of addition of rational numbers, if we add $\frac{2}{3}$ to $\frac{4}{3}$ we find that our sum is $\frac{6}{3}$.

Here is another illustration with rational numbers:



Note that $1 \longrightarrow \frac{1}{3}$. Therefore, by definition, going from the top scale to the bottom one is multiplication by $\frac{1}{3}$. To find the product of 5 and $\frac{1}{3}$, you locate 5 on the top scale, then look directly under it to find the answer ($\frac{5}{3}$).

$$5 \cdot \frac{1}{3} = \frac{5}{3}$$

Also note that point $\frac{1}{3}$ on the bottom scale goes to "1" on the top scale, so going from the bottom scale to the top scale is division by $\frac{1}{3}$. For example,

$$\frac{\frac{4}{3}}{\frac{1}{3}} = 4 \quad \text{and} \quad \frac{2}{\frac{1}{3}} = 6 \quad \text{and} \quad \frac{3}{\frac{1}{3}} = 9.$$

All of the mathematical operations discussed in this introduction are developed, step by step, throughout the unit. In order to maximize the understanding of this device, the children will, for the most part, construct their own scales.

The beginning lessons can be considered crucial and, therefore, should be slowly developed. If the understanding of the relationship between the scales is established early in the unit, the children are reasonably assured of success.

These methods have value in that they give us a way of picturing the different mathematical operations performed and, in addition, reveal certain relationships created by these operations. They are extremely powerful in that they work for addition, subtraction, multiplication, and division of any rational numbers (as well as for all other real numbers). A later unit will provide methods for actual computation.

The rolled tape which is suggested in some of the lessons refers to 2 - 3 1/2" adding machine tape. If each child is provided with a 2 foot strip he will be able to construct sufficiently long scales.

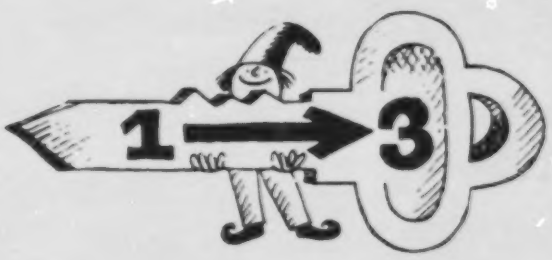
How to Construct a Number Scale

Draw two lines so that they do not meet. These are called parallel lines. Choose a segment to represent the Lilliputians' unit of measure. You may use a compass, ruler or paper marker. Slide this segment along the first line and mark each unit. Call the first point "0" and the second point "1". Continue to name the points along the line in order.

Directly under the Lilliputians' point "0" we begin to mark off these same units of measure on Gulliver's line. The zeros must match because zero units in Lilliput are the same length as zero units in Gulliver's measure. Both have no length.

Locate point "3" on the Lilliputians' scale. The matching point on Gulliver's scale is point "1". This is our key for the scale $1 \longrightarrow 3$. Put a box around the key.

Gulliver's point "1" goes to the Lilliputians' point "3". This gives us our key or operational clue for this scale. When we go from a point on Gulliver's scale to a corresponding point on the Lilliputians' scale we are performing the operation of multiplication by 3.



TEACHER'S MANUAL

Lesson One

Read to class.

One of the most fantastic adventure stories ever written was told by Jonathan Swift in the year 1726. "Gulliver's Travels" may be familiar to many of you. It is the story of a ship's surgeon who set out for the South Seas. During the journey his wooden ship hit a huge rock and was split. Captain Gulliver found himself alone in the raging sea. After swimming a distance, he reached an unfamiliar shore.

After some investigation he realized that he had found the small island of Lilliput and its tiny people, the Lilliputians. Naturally, because they were so small, their houses were small; their roads were narrow; their bridges, buildings, clothing and everything on their island were much smaller than anything Captain Gulliver had ever seen before.

In order for Captain Gulliver to get along on this unusual island, he had to learn many things. We are going to explore some of the Captain's problems that Jonathan Swift never mentioned in his book.

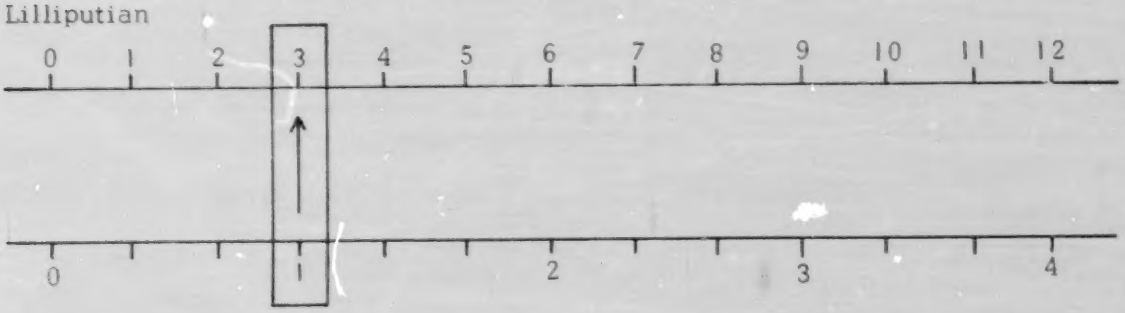
Because the Lilliputians were so tiny, they used a smaller unit of measure than Gulliver's people. We could construct a scale which would compare units of measure as they were used in Lilliput with those used in Gulliver's country.

Demonstrate on the chalkboard:

We draw two lines so that they do not meet. These are called parallel lines. You can use a compass, ruler or paper marker to choose a segment to represent the Lilliputians' unit of measure. Mark off this segment on the first line. Call the first point "0" and the second point "1". Slide this segment along the line and mark each unit. Continue to name the points along the line in order.

We know that one unit of measure in Gulliver's land was equal to three in the tiny Lilliputians' country. Directly under the Lilliputians' point "0" we begin to mark off these same units of measure on Gulliver's line. The zeros must match because zero units in Lilliput are the same length as zero units in Gulliver's measure. Both have no length.

Locate point "3" on the Lilliputian scale. The matching point on Gulliver's scale is point "1". This is our key for the scale $1 \longrightarrow 3$. Let us put a box around the key.



Note to Teacher:
Stress the understanding that any length can be chosen as a unit of measure for the Lilliputians. In addition, three of the Lilliputian units of measure are equal to one of Gulliver's. Each unit must be the exact same size. A compass, ruler or paper marker can accomplish this, if properly used.

Gulliver's point "1" goes to the Lilliputians' point "3". $(1 \longrightarrow 3)$ This gives us our key or operational clue for this scale. <u>When we go from a point on Gulliver's scale to a corresponding point on the Lilliputians' scale we are performing the operation of multiplication by 3</u>.

Let us look at the scale. Locate Gulliver's number 3. What is the corresponding number on the Lilliputians' scale? (9) We can write the mathematical sentence $3 \cdot 3 = 9$.

Directions to the children:
Part I

Now you may construct your own scales. Place Lesson One horizontally on your desk. Put your ruler under the directions of Part I. Draw two line segments across the page; one at the upper edge of your ruler; the other at the lower edge.

Note to Teacher:
Emphasize the need to draw parallel lines or lines which "will never meet".

Select the length of a unit of measure for the Lilliputians. You may choose any length. Slide this distance across the first line segment and mark each unit. Begin with zero and name each point in order.

Be sure to match the zero on Gulliver's line with the zero of the Lilliputian scale. Mark off each unit just as you did on the first line. Show Gulliver's point "1" matching the Lilliputian point "3". Put a box around the key $1 \longrightarrow 3$.

When you have completed your scale do the exercises on your worksheet.

Part 2

Use your number scale to complete the following:

$8 \cdot 3 = (24)$	$12 \cdot 3 = (36)$
$10 \cdot 3 = (30)$	$4 \cdot 3 = (12)$
$6 \cdot 3 = (18)$	$2 \cdot 3 = (6)$
$5 \cdot 3 = (15)$	$7 \cdot 3 = (21)$
$3 \cdot 3 = (9)$	$1 \cdot 3 = (3)$
$0 \cdot 3 = (0)$	$9 \cdot 3 = (27)$
$11 \cdot 3 = (33)$	

Part 3

Let us use "G" to stand for Gulliver's number and "L" to stand for the Lilliputians' number.

Refer to your number scale. Complete the following:

If G = 6, then L = (18).	If G = 4, then L = (12).
If G = 9, then L = (27).	If G = 0, then L = (0).
If G = 8, then L = (24).	If G = 5, then L = (15).
If G = 7, then L = (21).	If G = 2, then L = (6).

Part 4

The Lilliputians' number is equal to 3 times Gulliver's number. We can abbreviate it this way: $G \cdot 3 = L$.

Use your number scales to find the value of G.

If $G \cdot 3 = 21$

$G = (7)$

If $G \cdot 3 = 6$

$G = (2)$

If $G \cdot 3 = 12$

$G = (4)$

If $G \cdot 3 = 3$

$G = (1)$

If $G \cdot 3 = 9$

$G = (3)$

If $G \cdot 3 = 15$

$G = (5)$

If $G \cdot 3 = 18$

$G = (6)$

If $G \cdot 3 = 24$

$G = (8)$

If $G \cdot 3 = 0$

$G = (0)$

If $G \cdot 3 = 27$

$G = (9)$

Part 5

Gulliver knew it would be helpful to make a chart showing the relationship between the two scales. Let's help him to complete the chart.

G	L
2	(6)
4	(12)
1	(3)
12	(36)
8	(24)
10	(30)
3	(9)
0	(0)
7	(21)
5	(15)
6	(18)
9	(27)
11	(33)

Lesson Two

Directions to the children:
Part I

Refer to your number scale in Lesson One. When we went from a point on Gulliver's line to a corresponding point on the Lilliputians' line, we were multiplying by three.

Let's look at our key in this same scale as $3 \longrightarrow 1$. Point 3 on the Lilliputians' line corresponds to point "1" on Gulliver's line. This gives us division by three. When we go from a point on the Lilliputian scale to a matching point on Gulliver's scale, we are dividing by 3. We write it:

$$\frac{L}{3} = G \quad \text{or} \quad \frac{3}{3} = 1$$

Read: three divided by three equals one.

> Note to Teacher:
> We are indicating different ways of writing the same number. $\frac{9}{3}$ can be called by another name "3".

Look at your scale. Locate the Lilliputians' number 6. What is the corresponding number on Gulliver's line? (2) Let's write the mathematical sentence

$$\frac{6}{3} = 2.$$

Use your number scale to complete the following:

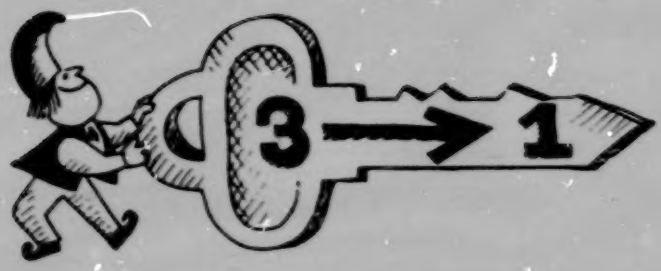
$\frac{9}{3} = (3)$ $\qquad$ $\frac{0}{3} = (0)$

$\frac{12}{3} = (4)$ $\qquad$ $\frac{6}{3} = (2)$

$\frac{3}{3} = (1)$ $\qquad$ $\frac{15}{3} = (5)$

$\frac{18}{3} = (6)$ $\qquad$ $\frac{21}{3} = (7)$

$\frac{24}{3} = (8)$



Part 2

Remember? We will use "G" to stand for Gulliver's number and "L" to stand for the Lilliputians' number.

Refer to your number scale to find the value of "G".

If L = 12, then G = (4).

If L = 18, then G = (6).

If L = 15, then G = (5).

If L = 24, then G = (8).

If L = 9, then G = (3).

Part 3

Can you give G another name?

$\frac{6}{3}$ = G (2)

$\frac{0}{3}$ = G (0)

$\frac{9}{3}$ = G (3)

$\frac{3}{3}$ = G (1)

$\frac{12}{3}$ = G (4)

Part 4

Let's show what we have learned. Complete this chart:

L	G
6	(2)
12	(4)
3	(1)
36	(12)
24	(8)
30	(10)
9	(3)
21	(7)
15	(5)
18	(6)
27	(9)
33	(11)

Directions to the children:

Compare the chart in Part 4 with the table in Lesson One, Part 5. What do you discover?

> Note to Teacher:
>
> The children should discover that when they multiply a number by three and then divide by three, they arrive at the original number.
>
> $4 \cdot 3 = 12$ $\qquad$ $\frac{12}{3} = 4$

When we multiply a number by three, then divide the answer by 3, we say we are performing an <u>inverse</u> operation. Multiplication is the inverse of division and vice versa.

For example:

If $2 \cdot 3 = 6$, then $\frac{6}{3} = 2$.

Show the inverse of the following:

If $2 \cdot 3 = 6$, then $(\frac{6}{3} = 2)$

If $9 \cdot 3 = 27$, then $(\frac{27}{3} = 9)$

If $5 \cdot 3 = 15$, then $(\frac{15}{3} = 5)$

If $\frac{24}{3} = 8$, then $(8 \cdot 3 = 24)$

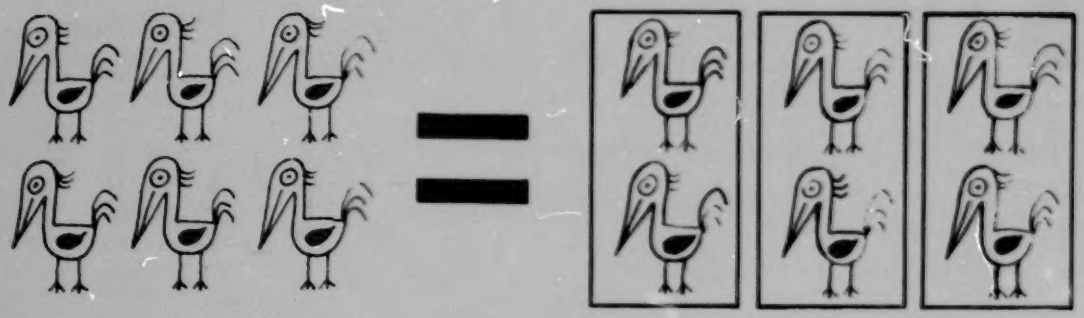
If $\frac{30}{3} = 10$, then $(10 \cdot 3 = 30)$

If $\frac{9}{3} = 3$, then $(3 \cdot 3 = 9)$

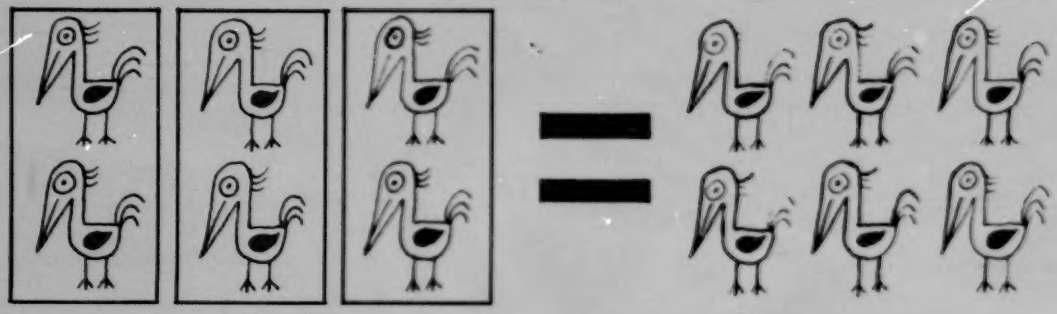
If $\frac{18}{3} = 6$, then $(6 \cdot 3 = 18)$

Here is another way to show an inverse operation.

$$\frac{6}{2} = 3$$



$$3 \cdot 2 = 6$$



Lesson Three

To the teacher:

Have children work in pairs. Provide one child with pint containers; the other with quart containers. Two plastic pails of the same size are also needed. Each child must have a tally sheet to record the number of pints he is pouring into his pail. Similarly, the child pouring quarts must also keep a record of quarts poured.

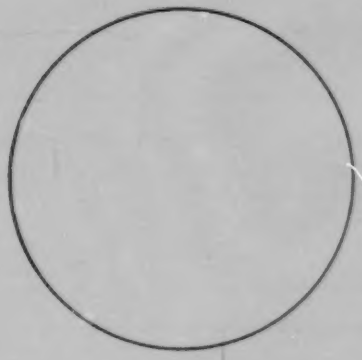
The object is for the two children working together to fill the pails to the same level. This can be determined by measuring the water level with a ruler or the pails could be marked for pints and quarts prior to the experiment.

When they have finished pouring they can then compare the number of pints it took to fill the pail with the number of quarts it took to reach that same level.

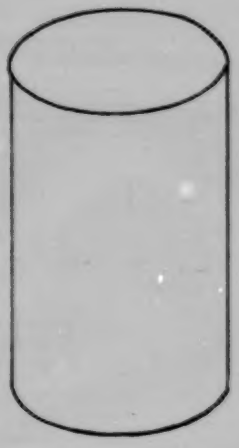
To the children:

Both Gulliver and the Lilliputians measured volume with graduated cylinders. In order to understand the difference between their measures Gulliver took two glass cylinders with a circular cross section:

Demonstrate on chalkboard:

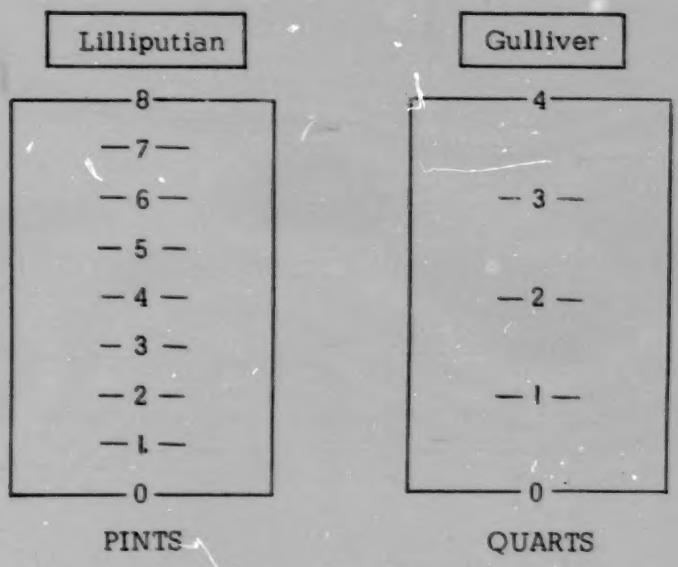


VIEW FROM ABOVE



VIEW FROM ANY SIDE

On each cylinder Gulliver drew a number line. The Lilliputians' unit of measure was pints. Gulliver's unit of measure was quarts.



Part I

Directions to the children:

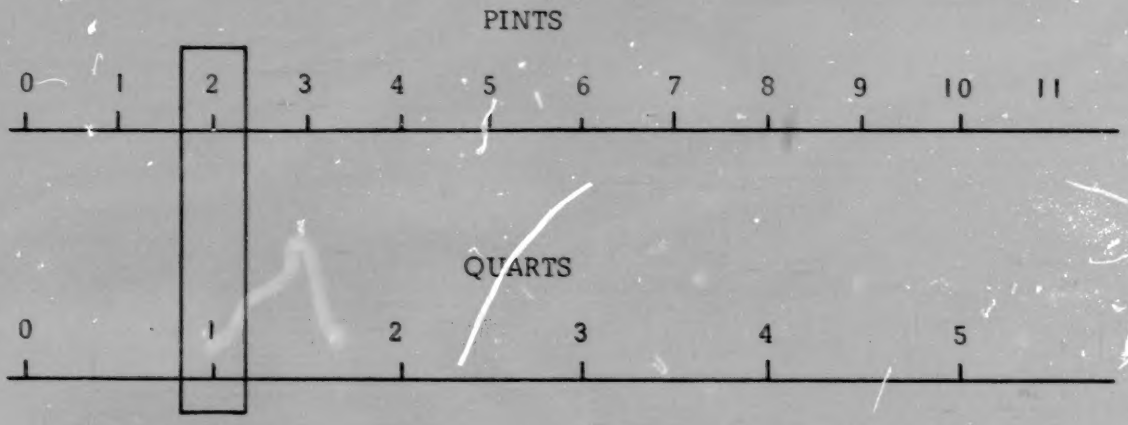
Construct a scale which will show the relationship between pints and quarts.

Use your ruler to draw two line segments. Draw one line with the upper edge; the other with the lower edge so that the line segments will not meet.

Select a short distance on your compass to represent pints. Slide this distance across the first line segment and mark each unit. Begin at zero and name the points in order.

Match zero of the "quart" line segment with zero of the pint segment. Mark off each unit along the quart line just as you did for the pints.

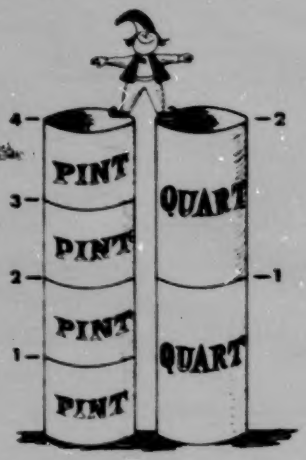
Our key for this scale is $2 \longrightarrow 1$ because there are 2 pints in one quart. We can also show it as a ratio of 2 : 1. Therefore, point two on the pint scale corresponds with point one on the quart scale. Put a box around the key.



Part 2

Gulliver thought it would be helpful to make a chart of the relationship between quarts and pints. Can you help him?

Quarts	Pints
12	(24)
6	(12)
5	(10)
3	(6)
8	(16)
0	(0)
2	(4)
9	(18)
11	(22)
4	(8)
1	(2)
7	(14)
10	(20)



We can show this chart in a short way. For example:

$$q \cdot 2 = p$$

$$4 \cdot 2 = 8$$

Refer to your chart and rewrite the chart using this short form.

$(12 \cdot 2 = 24)$ $(9 \cdot 2 = 18)$

$(6 \cdot 2 = 12)$ $(11 \cdot 2 = 22)$

$(5 \cdot 2 = 10)$ $(4 \cdot 2 = 8)$

$(3 \cdot 2 = 6)$ $(1 \cdot 2 = 2)$

$(8 \cdot 2 = 16)$ $(7 \cdot 2 = 14)$

$(0 \cdot 2 = 0)$ $(10 \cdot 2 = 20)$

$(2 \cdot 2 = 4)$

We could use another short form:

$$2q = p$$
$$2 \cdot 4 = 8$$

Refer to your chart and rewrite the chart in this form.

$(2 \cdot 12 = 24)$ $(2 \cdot 9 = 18)$

$(2 \cdot 6 = 12)$ $(2 \cdot 11 = 22)$

$(2 \cdot 5 = 10)$ $(2 \cdot 4 = 8)$

$(2 \cdot 3 = 6)$ $(2 \cdot 1 = 2)$

$(2 \cdot 8 = 16)$ $(2 \cdot 7 = 14)$

$(2 \cdot 0 = 0)$ $(2 \cdot 10 = 20)$

$(2 \cdot 2 = 4)$

Part 3

Now it should be easy to show the number of quarts we can make with pints.

Pints	Quarts
0	(0)
12	(6)
8	(4)
2	(1)
6	(3)
14	(7)
4	(2)
10	(5)
16	(8)

Can this chart be written in a short way? (Yes)
Like this:

$$q = \frac{\text{Number of pints}}{2}$$

$$q = \frac{6}{2}$$

$$3 = \frac{6}{2}$$

Refer to your chart and rewrite the examples using this short form.

$(6 = \frac{12}{2})$ $(7 = \frac{14}{2})$

$(4 = \frac{8}{2})$ $(2 = \frac{4}{2})$

$(1 = \frac{2}{2})$ $(5 = \frac{10}{2})$

$(3 = \frac{6}{2})$ $(8 = \frac{16}{2})$

Part 4

Find the unknown:

$4 \cdot 2 = p$ $q \cdot 2 = 20$

$p = (8)$ $q = (10)$

$6 \cdot 2 = p$ $q \cdot 2 = 16$

$p = (12)$ $q = (8)$

$q \cdot 2 = 12$ $q \cdot 2 = 14$

$q = (6)$ $q = (7)$

Lesson Four

When Gulliver had been in Lilliput for several months, his only suit of clothes became very worn. A friendly tailor offered to make a new suit for him. Gulliver told him that his suit would take about six yards of material. The Lilliputians, however, only measured in feet and did not understand yards as a measurement. Gulliver decided to construct a scale to help the tailor measure the cloth.

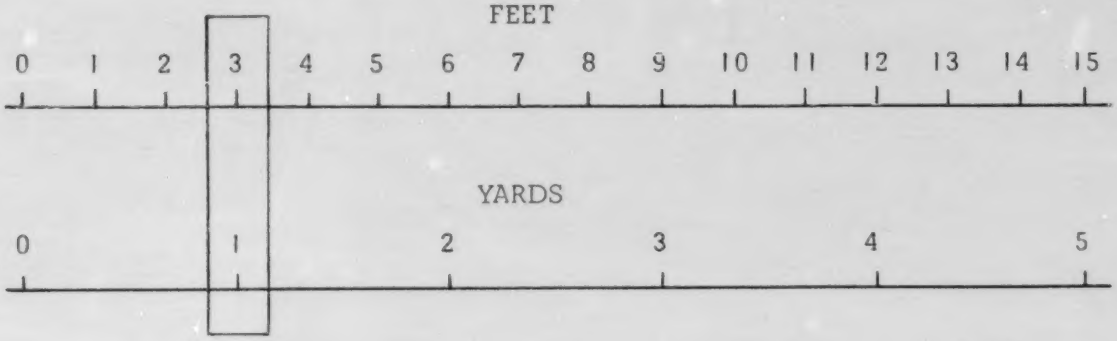
> Note to Teacher:
> It would be appropriate to reinforce the principle for constructing a number line scale for multiplication. That is if $1 \longrightarrow 3$. We are multiplying by 3; or we can say if $3 \longrightarrow 1$, we are dividing by 3.

Part I

Read to the class

We can construct the scales just as Gulliver did. Draw two line segments. Select any unit of measure and mark off the first line with that measure. Begin at zero and number the points above the line. This scale represents the Lilliputians' measure of feet.

Gulliver knew that there were 3 feet in a yard or 3 Lilliputians' measures to one Gulliver measure. (3 : 1) We can show this relationship by naming Gulliver's point "1" on the second line where the Lilliputians' point "3" matches. ($1 \longrightarrow 3$ represents multiplication by 3.) Put a box around key. Name the other points on our "yard line".



Example:

We know that Gulliver's suit took 6 yards of material. Locate 6 on Gulliver's number line. The tailor must use 18 feet since $6 \cdot 3 = 18$ feet.

Part 2

Let us call the number of feet "f" and the number of yards "y".

If y = 2, then f = (6).

If y = 4, then f = (12).

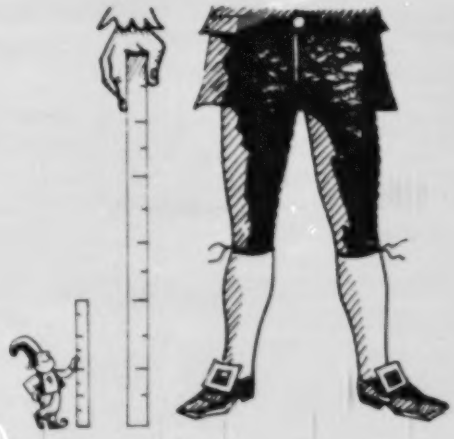
If y = 7, then f = (21).

If y = 3, then f = (9).

If y = 5, then f = (15).

If y = 6, then f = (18).

If y = 8, then f = (24).



Part 3

Can you help Gulliver complete this chart for the tailor?

y	f
3	(9)
0	(0)
8	(24)
6	(18)
11	(33)
9	(27)
1	(3)
7	(21)
2	(6)
5	(15)
4	(12)
10	(30)
12	(36)

Part 4

One day the Lilliputian tailor found a wide piece of cloth about 6 feet in length. He thought it might be made into a nice shirt for Gulliver. Gulliver said, "It will take two yards of material to make a shirt for me."

Look at your number scale. Will there be enough material? (Yes.) Write the number sentence like this: 6/3 = 2. Read: 6 divided by 3 is equal to 2.

If there were 9 feet of cloth, it would equal (3) yards. Write the number sentence like this: (9/3 = 3).

Complete the following:

If "f" = 9, then y = (3).

If "f" = 12, then y = (4).

If "f" = 3, then y = (1).

If "f" = 6, then y = (2).

If "f" = 18, then y = (6).

If "f" = 24, then y = (8).

Enrichment Activities

Why do we need feet and yards measures? Why not just use one unit of measure or the other?

Note to Teacher:

Designate children to work in pairs. One child is to measure with a foot ruler; the other with a yardstick.

1. Measure the length of the room. Compare measurement in feet and in yards. Which measure was more convenient for this purpose?

2. Measure the height of different children in the class. Which measure was more convenient for this purpose?

Here are other activities for comparison purposes:

3. Measure the length of the teacher's desk with foot rulers and yardsticks.

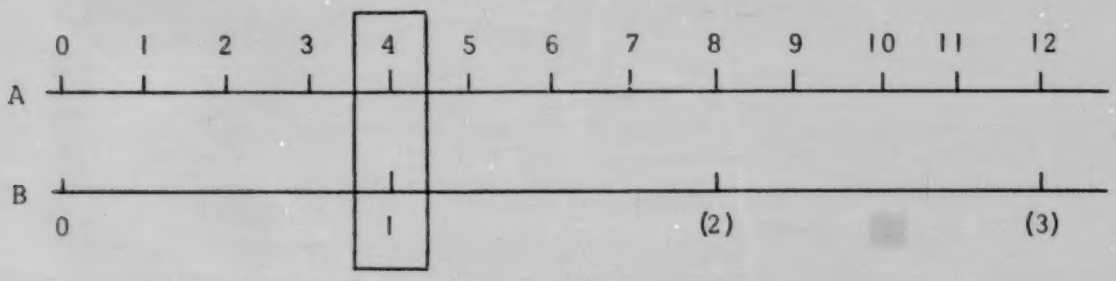
4. Measure bookshelves.

5. Measure the distance between pupils' desks.

Lesson Five

Part I

The key is $4 \longrightarrow 1$. Can you construct the scale? Use your tape.



What name will you give to the point on B which matches point 8 on scale A? (2) Can you name the point on B which matches point 12 on scale A? (3) point 16? (4)

Can you write the mathematical sentences for the points which you have matched? Do it!

$$\left(\frac{8}{4} = 2\right)$$

$$\left(\frac{12}{4} = 3\right)$$

$$\left(\frac{16}{4} = 4\right)$$

Suppose we look at the key this way $1 \longrightarrow 4$. Can you write these sentences another way? Try!

$$(2 \cdot 4 = 8)$$
$$(3 \cdot 4 = 12)$$
$$(4 \cdot 4 = 16)$$

Extend your scale. Write additional mathematical sentences.

$(5 \cdot 4 = 20)$ $(7 \cdot 4 = 28)$ $(9 \cdot 4 = 36)$ $(11 \cdot 4 = 44)$

$(6 \cdot 4 = 24)$ $(8 \cdot 4 = 32)$ $(10 \cdot 4 = 40)$ $(12 \cdot 4 = 48)$

Note to Teacher:

Please refer to Lesson Three, Part I for the preliminary activity for this lesson. Substitute gallon containers for the pint containers. The children should discover the relationship between quarts and gallons. The relationship between pints and gallons may be recognized by the children. However, it will only be significant learning if it comes from them. For example: 8 pints = 1 gallon.

Part 2

You have discovered that there are four quarts in one gallon. Use this scale with the key $1 \longrightarrow 4$ to find the value of g.

g = gallons

q = quarts

If $g \cdot 4 = 12$

$g = (3)$

If $g \cdot 4 = 20$

$g = (5)$

If $g \cdot 4 = 4$

$g = (1)$

If $g \cdot 4 = 16$

$g = (4)$

If $g \cdot 4 = 8$

$g = (2)$

If $g \cdot 4 = 40$

$g = (10)$

If $g \cdot 4 = 36$

$g = (9)$

If $g \cdot 4 = 32$

$g = (8)$

If $g \cdot 4 = 28$

$g = (7)$

If $g \cdot 4 = 24$

$g = (6)$

Part 3

In the last exercise we <u>multiplied</u> the number of <u>gallons</u> by four in order to arrive at the same amount as measured by quarts.

To perform the inverse operation we <u>divide</u> the number of quarts by four in order to arrive at the same amount as measured by gallons. We can write this in a short form: If $g \cdot 4 = q$, then $\frac{q}{4} = g$.

Find the number of quarts which must be divided by four in order for the mathematical sentence to be correct.

If $\frac{q}{4} = 1$

$q = (4)$

If $\frac{q}{4} = 2$

$q = (8)$

If $\frac{q}{4} = 5$

$q = (20)$

If $\frac{q}{4} = 6$

$q = (24)$

If $\frac{q}{4} = 8$

$q = (32)$

If $\frac{q}{4} = 9$

$q = (36)$

If $\frac{q}{4} = 10$

$q = (40)$

If $\frac{q}{4} = 3$

$q = (12)$

What do you notice about the problems in this lesson?

Note to Teacher:

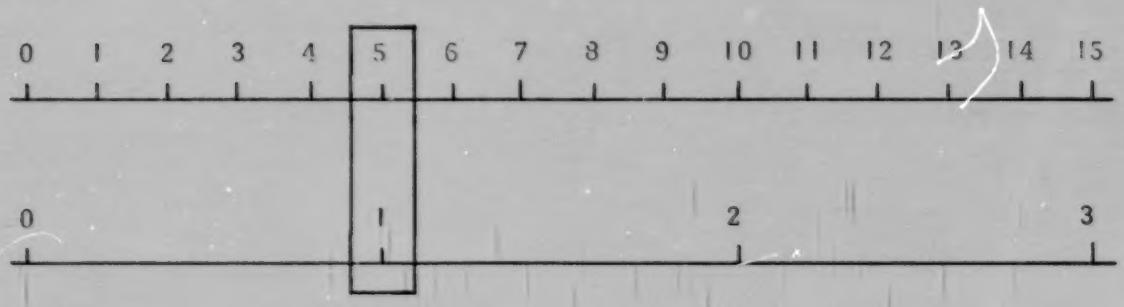
The children should recognize that when they multiply by a given number and then divide by that number, they return to their original number. In mathematical terms we say that division is the inverse of multiplication and vice versa. Refer to Lesson Two for a review of this concept, if necessary.

Lesson Six

Part 1

Gulliver had a money problem too. The Lilliputians used only pennies; five of their pennies were equivalent to one of Gulliver's nickels. Let's make a scale which will compare the number of pennies with the number of nickels. What will be our key? (5 ⟶ 1). Construct the scale on your tape. Put a box around the key.

> Note to Teacher:
> The children should be able to construct the number scale on their own, according to the method we've been using in this unit.



Part 2

Let's make a table which shows a comparison between pennies and nickels. Let P stand for pennies and let N stand for nickels. Use your number scale.

N	P
4	(20)
0	(0)
10	(50)
3	(15)
1	(5)
7	(35)
9	(45)
2	(10)
5	(25)
6	(30)
8	(40)
12	(60)
11	(55)

Part 3

Complete the following:

1 nickel = 5 pennies or $1 \cdot 5 = 5$

4 nickels = (20) pennies or $(4 \cdot 5 = 20)$

9 nickels = (45) pennies or $(9 \cdot 5 = 45)$

3 nickels = (15) pennies or $(3 \cdot 5 = 15)$

6 nickels = (30) pennies or $(6 \cdot 5 = 30)$

8 nickels = (40) pennies or $(8 \cdot 5 = 40)$

5 nickels = (25) pennies or $(5 \cdot 5 = 25)$

2 nickels = (10) pennies or $(2 \cdot 5 = 10)$

Lesson Seven

Read to the children:

Bankers Exchange

If the Lilliputians were to visit with Gulliver when he returned to his homeland they would probably find it more convenient to exchange many of their pennies for nickels.

We can set up our own exchange for that purpose.

> Note to Teacher:
> Designate half the class as Bankers; the other half as Lilliputians. It is advisable to have alternating rows of Bankers and Lilliputians. Provide red and blue counters.

If you are a Lilliputian you should have all the red counters which will represent pennies. If you are a Banker you should have all the blue counters which will represent the nickels.

How many pennies must a Lilliputian give to a Banker in order to get one nickel? (5) Make the exchange.

Write the number sentence for the exchange on your worksheet. The Lilliputian will write $\frac{5}{5} = 1$.

The Banker will write $1 \cdot 5 = 5$. Make as many exchanges with your partner as you can and write the sentence for each exchange.

Possible exchanges:

($\frac{5}{5} = 1$)

($\frac{10}{5} = 2$)

($\frac{15}{5} = 3$)

($\frac{20}{5} = 4$)

($\frac{25}{5} = 5$)

($\frac{30}{5} = 6$)

($\frac{35}{5} = 7$)

($1 \cdot 5 = 5$)

($2 \cdot 5 = 10$)

($3 \cdot 5 = 15$)

($4 \cdot 5 = 20$)

($5 \cdot 5 = 25$)

($6 \cdot 5 = 30$)

($7 \cdot 5 = 35$)

Lesson Eight

Read to class:

Gulliver found it difficult to get enough food to satisfy his appetite. He discovered that six Lilliputian apples equaled one average sized apple in weight. Of course, he understood that tiny people eat tiny apples.

Part 1

Construct the scale using the key $6 \longleftrightarrow 1$. Use your tape.

> Note to Teacher:
> Provide the rolled tape so that the number scale can be extended to include an adequate number of multiples of six.

Part 2

Let L represent the number of Lilliputian apples which Gulliver ate. Let G represent the equivalent number of apples Gulliver would have eaten in his own country.

If G = 3, then L = (18)	If L = 18, then G = (3)
If G = 5, then L = (30)	If L = 30, then G = (5)
If G = 4, then L = (24)	If L = 24, then G = (4)
If G = 7, then L = (42)	If L = 42, then G = (7)
If G = 2, then L = (12)	If L = 12, then G = (2)
If G = 6, then L = (36)	If L = 36, then G = (6)

Part 3

Can you write the mathematical sentences for the statements in Part 2? Let's start with the first one:

If G = 3, then L = 18

or

$3 \cdot 6 = 18$

Do the rest of them in the same way.

	$(\frac{18}{6} = 3)$
$(5 \cdot 6 = 30)$	$(\frac{30}{6} = 5)$
$(4 \cdot 6 = 24)$	$(\frac{24}{6} = 4)$
$(7 \cdot 6 = 42)$	$(\frac{42}{6} = 7)$
$(2 \cdot 6 = 12)$	$(\frac{12}{6} = 2)$
$(6 \cdot 6 = 36)$	$(\frac{36}{6} = 6)$

Part 4

Write some of the mathematical sentences you know which show multiplication by 6. For example:

$1 \cdot 6 = 6$	$(7 \cdot 6 = 42)$
$(2 \cdot 6 = 12)$	$(8 \cdot 6 = 48)$
$(3 \cdot 6 = 18)$	$(9 \cdot 6 = 54)$
$(4 \cdot 6 = 24)$	$(10 \cdot 6 = 60)$
$(5 \cdot 6 = 30)$	$(11 \cdot 6 = 66)$
$(6 \cdot 6 = 36)$	$(12 \cdot 6 = 72)$

Part 5

Use your scale to solve the following problems:

$6 \cdot 0 = (0)$	$0 \cdot 6 = (0)$
$6 \cdot 9 = (54)$	$9 \cdot 6 = (54)$
$7 \cdot 6 = (42)$	$6 \cdot 7 = (42)$
$4 \cdot 6 = (24)$	$6 \cdot 4 = (24)$
$6 \cdot 8 = (48)$	$8 \cdot 6 = (48)$
$1 \cdot 6 = (6)$	$6 \cdot 1 = (6)$

$6 \cdot 5 = (30)$	$5 \cdot 6 = (30)$
$6 \cdot 3 = (18)$	$3 \cdot 6 = (18)$
$10 \cdot 6 = (60)$	$6 \cdot 10 = (60)$
$6 \cdot 2 = (12)$	$2 \cdot 6 = (12)$

What do you notice about the above problems?

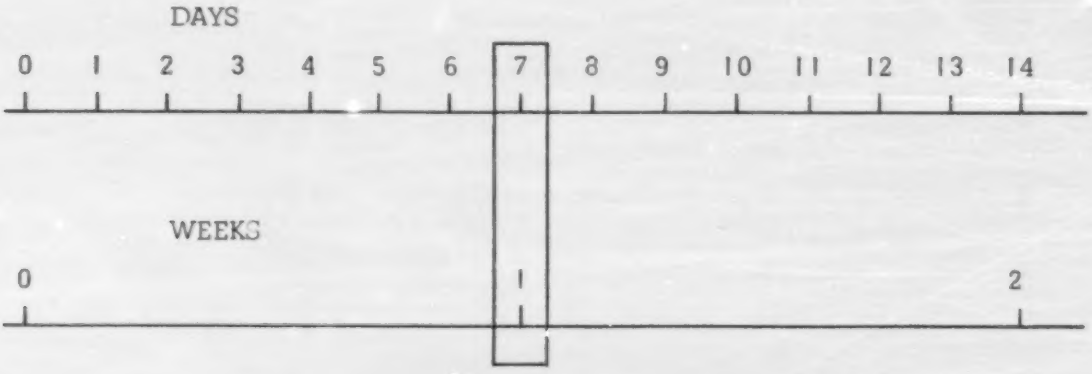
Note to Teacher:

Reinforce commutative law. If we start with 4 and apply the multiplication by 2 operation we get the same answer as if we started with 2 and applied the multiplication by 4 operation. (See Teacher's Introd., Unit 17)

Lesson Nine

Part I

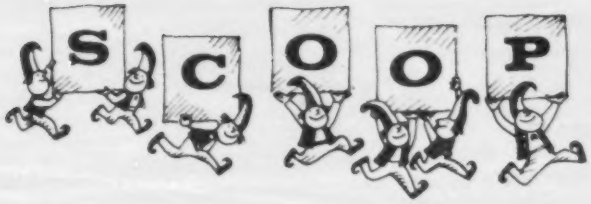
Suppose we wanted to find out the number of days there are in three weeks. Can we construct a number scale to help us? (Yes.) What will our key be? (7 ⟶ 1). Construct the scale on your tape.



Part 2

Complete the following:

3 weeks = (21) days	56 days = (8) weeks
1 week = (7) days	14 days = (2) weeks
7 weeks = (49) days	63 days = (9) weeks
5 weeks = (35) days	49 days = (7) weeks
9 weeks = (63) days	35 days = (5) weeks



Part 3

LET'S PLAY SCOOP

> Note to Teacher:
> Designate partners for the game of SCOOP. Read the directions below to the children.

Directions for SCOOP

Each person is to write four mathematical sentences for multiplication by 7. The sentences may be in any order. Do not include the answers.

Trade lists with your partner. See who can be the first to complete the sentences correctly. The winner of the first match is awarded an S.

Now write four additional sentences for multiplication by 7. Exchange again. The winner of each match earns another letter. The game is won when the first person has been awarded all the letters in SCOOP.

Play the game again using mathematical sentences which show division by 7.

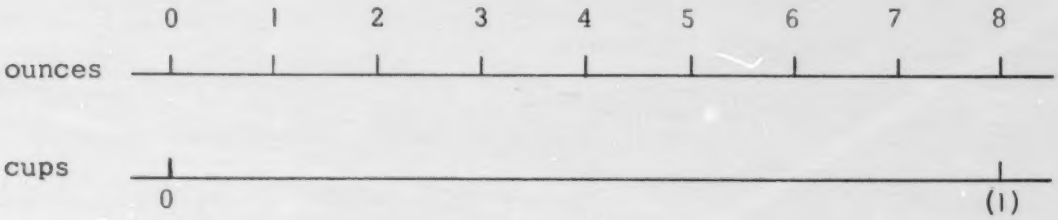
Lesson Ten

Part 1

A healthful diet was important to our Lilliputian friends. The Department of Health recommended that each Lilliputian child drink eight ounces of milk each day.

Although Gulliver knew very little about farming he thought it might be fun to work on a dairy farm for a change. Off he went with his shiny measuring cup. His job was to pour the milk into one-ounce (oz.) containers.

Can you construct a number scale which would show the number of containers he could fill with one measuring cupful of milk? Because there are eight ounces (ozs.) in one cup our key to the scale would be eight to one or $8 \longrightarrow 1$. Use your tape for this scale.



Part 2

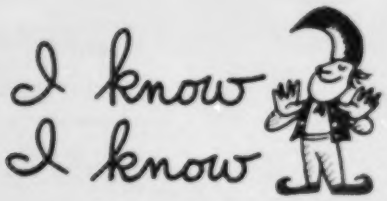
For every one cup we can have 8 one-ounce containers. A simple way to write it is $1 \cdot 8 = 8$.

Use your scale to discover the number of one-ounce containers Gulliver can fill with 5 cups; 6 cups; 4 cups; 2 cups; 3 cups.

Part 3

Can you fill in the boxes?

☐'s	1	2	3	4	5	6	7	8	9	10	11	12
☐'s ozs.	8	(16)	(24)	(32)	(40)	(48)	(56)	(64)	(72)	(80)	(88)	(96)



Part 4

Who Knows? I KNOW

Read directions to children:

Work with a partner. Each partner should have a tally sheet. Choose someone to be the leader. The leader gives an answer to a multiplication or division problem involving eight. If the partner can write the correct mathematical sentence, either multiplication or division, he may write the letter "I" on his tally sheet.

If he can express the answer as both multiplication and division sentences he may begin two sentences by placing the second I on the line below the first.

Now let the other person be the leader.

Each time the mathematical sentence is written correctly another letter may be written. The first person to write I KNOW 2 times is the winner.

Lesson Eleven

> Note to Teacher:
> Thus far, we have been demonstrating multiplication on the number scales. In this lesson we will use another approach. It is assumed, however, that the children know how to relate multiplication by nine to the number scales. The children will require additional paper for this exercise.

Part 1

The most popular flower in Lilliput had nine petals.

How many petals are there on two Lilliputian flowers? (18)

Can you express this as a mathematical sentence? ($2 \cdot 9 = 18$)

Can you draw the number of flowers which will show 36 petals? Do it!

Draw the number of Lilliputian flowers that show 27 petals; 63 petals; 81 petals; 45 petals; 72 petals; 54 petals; 90 petals; 99 petals.

Part 2

Write the mathematical sentence for each set of drawings in Part 1.

Let p = petals

Let f = flowers

then

$$\frac{p}{9} = f$$

For example: $\frac{27}{9} = 3$

$(\frac{63}{9} = 7)$

$(\frac{81}{9} = 9)$

$(\frac{45}{9} = 5)$

$(\frac{72}{9} = 8)$

$(\frac{54}{9} = 6)$

$(\frac{90}{9} = 10)$

$(\frac{99}{9} = 11)$

Part 3

Rewrite these drawings as multiplication sentences: $f \cdot 9 = p$

$(7 \cdot 9 = 63)$

$(9 \cdot 9 = 81)$

$(5 \cdot 9 = 45)$

$(8 \cdot 9 = 72)$

$(6 \cdot 9 = 54)$

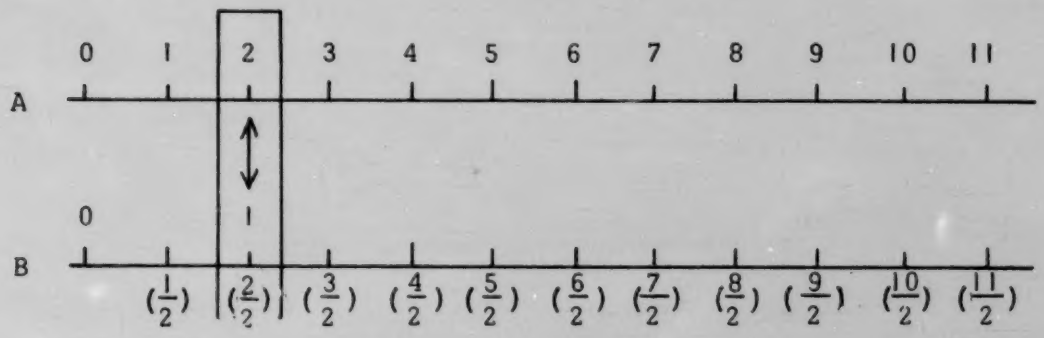
$(10 \cdot 9 = 90)$

$(11 \cdot 9 = 99)$

Lesson Twelve

Part I

Let's examine the scale in this lesson together.



What does the key tell us?

1. That when we go from a point on B to a matching point on A, we are multiplying by two.
2. That when we go from a point on A to a matching point on B, we are dividing by two.

We have learned that we can call a point by different names. What other name could we give point "1" on B? $(\frac{2}{2})$

Yes, we could name this according to the operation we have performed, i.e., two divided-by-two.

Note to Teacher:

It is necessary to emphasize that there are many names for the same number $(\frac{2}{4}, \frac{1}{2}, \frac{-2}{-4})$ and all represent the same number. For convenience purposes we usually choose the simplest expression. This is particularly important when we begin to add fractions with different denominators. Fractions are easier to add when they have the same denominators, i.e., they can be located on the same number line scale. When we wish to add $\frac{1}{2} + \frac{1}{3}$ we may choose the name $\frac{6}{12}$ for $\frac{1}{2}$ and $\frac{4}{12}$ for $\frac{1}{3}$. Both fractions can be located on the scale which has twelfths.

What name can we give to the point on B which matches point "1" on A? ($\frac{1}{2}$)

Yes, we can call it by the operation we perform, one divided by two.

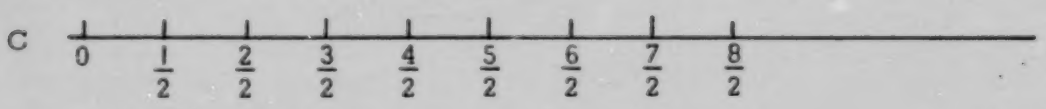
Can you name the other points on "B"? Try!

Part 2

Can we add $\frac{1}{2} + \frac{1}{2}$ on this number scale?

> Note to Teacher:
> Allow children time to respond. The need for two scales may be suggested by them. It may also be necessary to review the method for addition of whole numbers using the number line (sliding to the right).

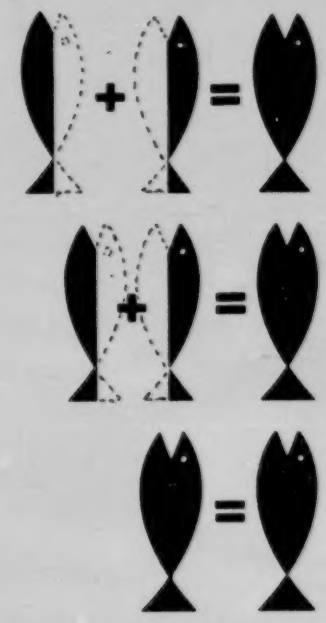
Yes, we must construct our sliding scale. It must be just like our B scale in Part 1. Call it scale C.



Tear off your sliding scale and add $\frac{1}{2} + \frac{1}{2}$. We simply locate point $\frac{1}{2}$ on B. Match zero of the sliding scale C to that point. Locate $\frac{1}{2}$ on C. We read our answer on B: $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$.

> Note to Teacher:
> It has been common in our language to refer to fractions such as $\frac{3}{2}$, $\frac{5}{3}$, or $\frac{4}{4}$ as "improper fractions." We are told to express them as mixed numbers (a whole number and a fraction, i.e., $\frac{5}{3}$ as $1\frac{2}{3}$). The word "improper" is a misnomer. Mathematically a fraction whose numerator is equal to or greater than its denominator is "proper." For example, it would be impossible to express $\frac{6}{\pi}$ as a mixed number.

Here is another way to show $\frac{1}{2} + \frac{1}{2}$.



Part 3

Use your number scales to add the following:

$\frac{2}{2} + \frac{3}{2} = (\frac{5}{2})$

$\frac{4}{2} + \frac{2}{2} = (\frac{6}{2})$

$\frac{1}{2} + \frac{5}{2} = (\frac{6}{2})$

$\frac{3}{2} + \frac{5}{2} = (\frac{8}{2})$

$\frac{5}{2} + \frac{1}{2} = (\frac{6}{2})$

$\frac{6}{2} + \frac{3}{2} = (\frac{9}{2})$

$\frac{7}{2} + \frac{2}{2} = (\frac{9}{2})$

$\frac{8}{2} + \frac{1}{2} = (\frac{9}{2})$

$\frac{8}{2} + \frac{2}{2} = (\frac{10}{2})$

$\frac{1}{2} + \frac{4}{2} = (\frac{5}{2})$

Part 4

Discover Y's name: (Solve for "Y".) Use your number scales.

$\frac{4}{2} + \frac{y}{2} = \frac{7}{2}$ $\quad y = (3)$

$\frac{3}{2} + \frac{y}{2} = \frac{9}{2}$ $\quad y = (6)$

$\frac{1}{2} + \frac{y}{2} = \frac{5}{2}$ $\quad y = (4)$

$\frac{5}{2} + \frac{y}{2} = \frac{11}{2}$ $\quad y = (6)$

$\frac{7}{2} + \frac{y}{2} = \frac{14}{2}$ $\quad y = (7)$

Part 5

Can we use these number scales to subtract fractions? If so, complete the following:

$\frac{8}{2} - \frac{3}{2} = (\frac{5}{2})$ $\qquad$ $\frac{7}{2} - \frac{3}{2} = (\frac{4}{2})$

$\frac{9}{2} - \frac{1}{2} = (\frac{8}{2})$ $\qquad$ $\frac{6}{2} - \frac{4}{2} = (\frac{2}{2})$

$\frac{5}{2} - \frac{2}{2} = (\frac{3}{2})$ $\qquad$ $\frac{10}{2} - \frac{3}{2} = (\frac{7}{2})$

Lesson Thirteen

Part 1

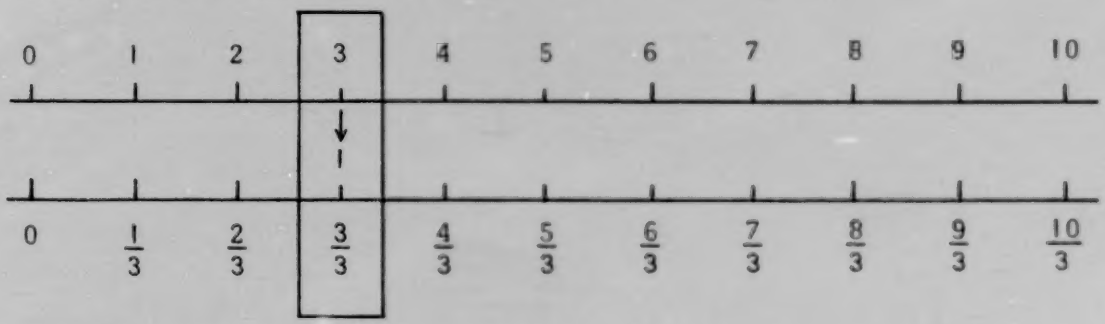
To the children: (Demonstrate, if necessary)

We have constructed a helpful device to add halves. Suppose we wanted to add thirds? Let us see if we can construct a number scale to help us.

Draw two number lines. Mark off the points on A and B at equal distances. Name the points on A. Begin with zero. Remember "zero" on A and B must match.

> Note to Teacher:
> Give children time to discover the procedure for completing the scale.

Yes, our key will be $3 \longrightarrow 1$. Name point "1" on B. Put a box around these points.



This means we locate the point on our second scale which corresponds to 3 on the first scale. What new name can we give this point? $(\frac{3}{3})$

Can you name the other points on the second scale? Try it!

Part 2

If you wanted to add $\frac{1}{3} + \frac{4}{3}$ would the scale which we've constructed help us? (Yes, but we'd need another one which is exactly the same.)

All right, let's construct a sliding scale showing thirds. Remember, it must be the same as scale B.

Part 3

Use your number scale to add these fractions:

$\frac{2}{3} + \frac{3}{3} = (\frac{5}{3})$ $\quad\quad$ $\frac{2}{3} + \frac{5}{3} = (\frac{7}{3})$

$\frac{4}{3} + \frac{5}{3} = (\frac{9}{3})$ $\quad\quad$ $\frac{5}{3} + \frac{3}{3} = (\frac{8}{3})$

$\frac{6}{3} + \frac{2}{3} = (\frac{8}{3})$ $\quad\quad$ $\frac{3}{3} + \frac{2}{3} = (\frac{5}{3})$

$\frac{1}{3} + \frac{4}{3} = (\frac{5}{3})$ $\quad\quad$ $\frac{1}{3} + \frac{5}{3} = (\frac{6}{3})$

Can you find s?

$\frac{5}{3} - \frac{s}{3} = \frac{1}{3}$ $\quad\quad$ $s = (4)$

$\frac{8}{3} - \frac{s}{3} = \frac{5}{3}$ $\quad\quad$ $s = (3)$

$\frac{9}{3} - \frac{s}{3} = \frac{1}{3}$ $\quad\quad$ $s = (8)$

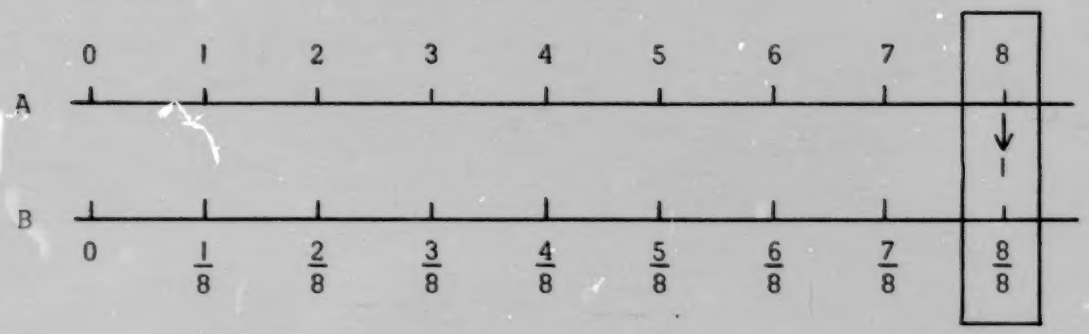
$\frac{7}{3} - \frac{s}{3} = \frac{2}{3}$ $\quad\quad$ $s = (5)$

$\frac{6}{3} - \frac{s}{3} = \frac{5}{3}$ $\quad\quad$ $s = (1)$

Lesson Fourteen

Part I

Naming points can be fun. Here are two number scales. Can you name the points on B?



$8 \longrightarrow 1$. We, therefore, have scale B showing division by 8.

Part 2

Could we add $\frac{3}{8} + \frac{2}{8}$?

Yes, we need only to construct another scale exactly like B which we could slide. Do it! Then add the following on your scale:

$\frac{1}{8} + \frac{6}{8} = (\frac{7}{8})$ $\quad$ $\frac{3}{8} + \frac{4}{8} = (\frac{7}{8})$

$\frac{1}{8} + \frac{3}{8} = (\frac{4}{8})$ $\quad$ $\frac{4}{8} + \frac{3}{8} = (\frac{7}{8})$

$\frac{2}{8} + \frac{7}{8} = (\frac{9}{8})$ $\quad$ $\frac{6}{8} + \frac{2}{8} = (\frac{8}{8})$

$\frac{5}{8} + \frac{1}{8} = (\frac{6}{8})$ $\quad$ $\frac{3}{8} + \frac{1}{8} = (\frac{4}{8})$

Part 3

Solve for w:

If $\frac{8}{8} - \frac{w}{8} = \frac{4}{8}$, then w = (4)

If $\frac{7}{8} - \frac{w}{8} = \frac{3}{8}$, then w = (4)

If $\frac{11}{8} - \frac{w}{8} = \frac{8}{8}$, then w = (3)

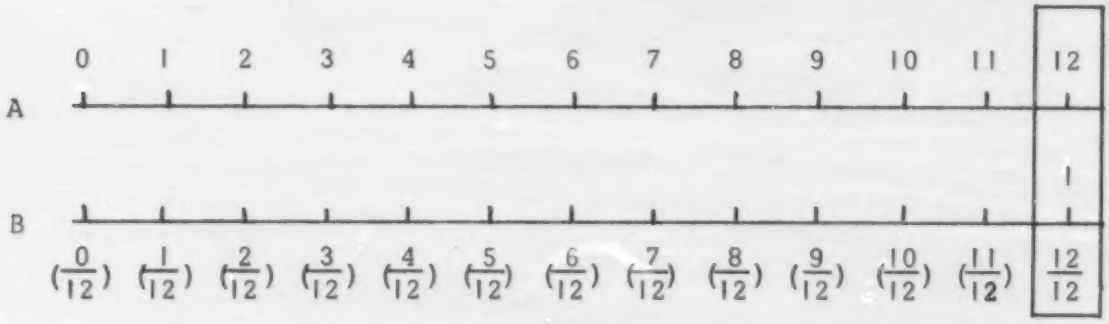
If $\frac{6}{8} - \frac{w}{8} = \frac{1}{8}$, then w = (5)

If $\frac{5}{8} - \frac{w}{8} = 0$, then w = (5)

Lesson Fifteen

Part I

Let's try our skill on another scale. Name the points on B. Cut out the sliding scale at the bottom of the page.



Note to Teacher:
Designate odd and even rows.

Odd numbered rows: Add fractions in Column I.

Even numbered rows: Add fractions in Column II.

I	II
$\frac{1}{12} + \frac{5}{12} = (\frac{6}{12})$	$\frac{5}{12} + \frac{1}{12} = (\frac{6}{12})$
$\frac{2}{12} + \frac{9}{12} = (\frac{11}{12})$	$\frac{9}{12} + \frac{2}{12} = (\frac{11}{12})$
$\frac{5}{12} + \frac{7}{12} = (\frac{12}{12})$	$\frac{7}{12} + \frac{5}{12} = (\frac{12}{12})$
$\frac{0}{12} + \frac{9}{12} = (\frac{9}{12})$	$\frac{9}{12} + \frac{0}{12} = (\frac{9}{12})$
$\frac{11}{12} + \frac{3}{12} = (\frac{14}{12})$	$\frac{3}{12} + \frac{11}{12} = (\frac{14}{12})$

I	II
$\frac{6}{12} + \frac{4}{12} = (\frac{10}{12})$	$\frac{4}{12} + \frac{6}{12} = (\frac{10}{12})$
$\frac{8}{12} + \frac{3}{12} = (\frac{11}{12})$	$\frac{3}{12} + \frac{8}{12} = (\frac{11}{12})$
$\frac{7}{12} + \frac{8}{12} = (\frac{15}{12})$	$\frac{8}{12} + \frac{7}{12} = (\frac{15}{12})$

Note to Teacher:
The children should discover the need to continue to name the points beyond $\frac{12}{12}$.

Compare your answers with those of your partner. What do you discover?

Note to Teacher:
Reinforce the commutative principle.

Solve for b:

$\frac{9}{12} + \frac{b}{12} = \frac{11}{12}$ $\quad b = (2)$

$\frac{3}{12} + \frac{b}{12} = \frac{9}{12}$ $\quad b = (6)$

$\frac{8}{12} + \frac{b}{12} = \frac{12}{12}$ $\quad b = (4)$

$\frac{6}{12} + \frac{b}{12} = \frac{10}{12}$ $\quad b = (4)$

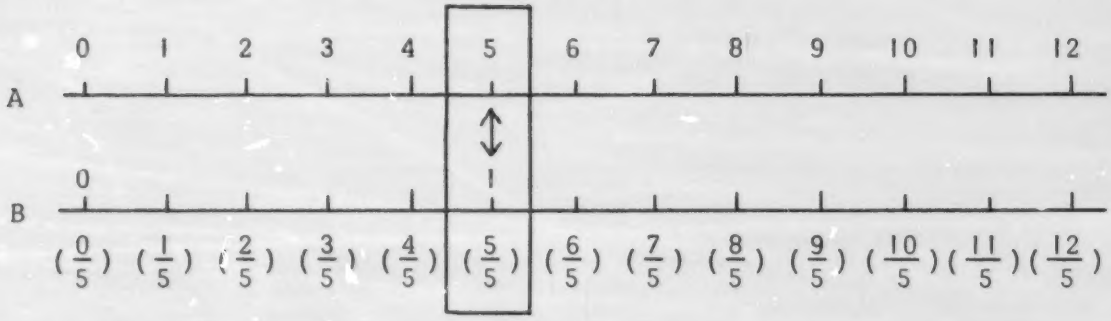
$\frac{1}{12} + \frac{b}{12} = \frac{8}{12}$ $\quad b = (7)$

$\frac{4}{12} + \frac{b}{12} = \frac{9}{12}$ $\quad b = (5)$

Lesson Sixteen

Part I

Name the points on B scale.



Demonstrate with children:

Our key shows us that we can multiply and divide by five. Point 5 on A scale goes to point 1 on B scale. Therefore, going from a point on A to a matching point on B gives us division by 5.

We can look at this scale another way: $1 \longrightarrow 5$. Therefore, when we go from a point on B scale to a corresponding point on A scale we are multiplying by 5.

For example, point $\frac{6}{5}$ on B scale multiplied by five gives us the answer of "six" on A scale. $(\frac{6}{5} \cdot 5 = 6)$.

Use this number scale to find the answers to the following problems. Extend the scale, if necessary.

$\frac{8}{5} \cdot 5 = (8)$ $\qquad$ $\frac{5}{5} \cdot 5 = (5)$

$\frac{2}{5} \cdot 5 = (2)$ $\qquad$ $\frac{1}{5} \cdot 5 = (1)$

$\frac{11}{5} \cdot 5 = (11)$ $\qquad$ $\frac{9}{5} \cdot 5 = (9)$

$\frac{3}{5} \cdot 5 = (3)$ $\qquad$ $\frac{4}{5} \cdot 5 = (4)$

$\frac{15}{5} \cdot 5 = (15)$ $\qquad$ $\frac{7}{5} \cdot 5 = (7)$

$\frac{13}{5} \cdot 5 = (13)$ $\quad$ $\frac{6}{5} \cdot 5 = (6)$

$\frac{16}{5} \cdot 5 = (16)$ $\quad$ $\frac{10}{5} \cdot 5 = (10)$

Part 2

Find the value of b:

$\frac{b}{5} \cdot 5 = 7$ $\quad$ $b = (7)$ $\quad$ $\frac{b}{5} \cdot 5 = 1$ $\quad$ $b = (1)$

$\frac{b}{5} \cdot 5 = 11$ $\quad$ $b = (11)$ $\quad$ $\frac{b}{5} \cdot 5 = 9$ $\quad$ $b = (9)$

$\frac{b}{5} \cdot 5 = 3$ $\quad$ $b = (3)$ $\quad$ $\frac{b}{5} \cdot 5 = 6$ $\quad$ $b = (6)$

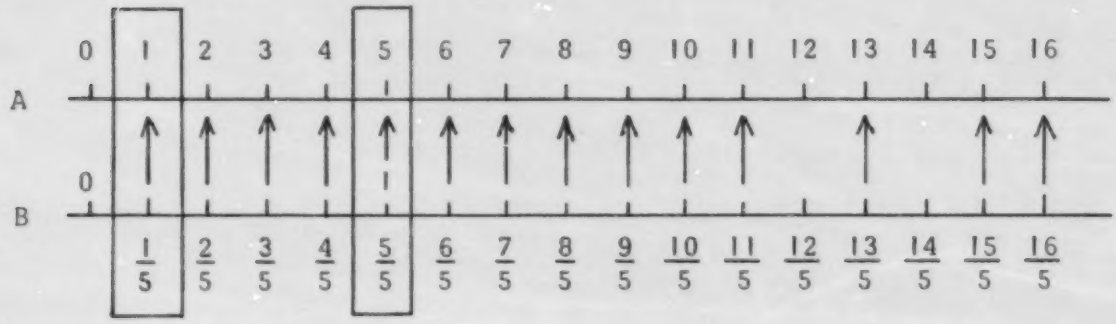
$\frac{b}{5} \cdot 5 = 10$ $\quad$ $b = (10)$ $\quad$ $\frac{b}{5} \cdot 5 = 8$ $\quad$ $b = (8)$

$\frac{b}{5} \cdot 5 = 2$ $\quad$ $b = (2)$ $\quad$ $\frac{b}{5} \cdot 5 = 4$ $\quad$ $b = (4)$

Lesson Seventeen

Part I

What other interesting things can we discover about this scale?



Note to Teacher:
Give children time to discover that $1 \longrightarrow \frac{1}{5}$ and $\frac{1}{5} \longrightarrow 1$.
This relationship gives us another scale within our basic $5 \longleftrightarrow 1$ scale.

Demonstrate with children:

Remember? Whatever $1 \longrightarrow$ (goes to) is multiplication by that number. Also, whatever (goes to) $\longrightarrow 1$, gives us a scale which shows division by that number.

Yes, our scale also shows that $\frac{1}{5} \longrightarrow 1$. This gives us another key. Therefore, when we go from a point on B to a matching point on A we can look at it as division by $\frac{1}{5}$. Put a box around the second key $\frac{1}{5} \longrightarrow 1$.

For example, point $\frac{6}{5}$ on B scale divided by $\frac{1}{5} = 6$ as shown on A scale.

$$\text{(written } \frac{\frac{6}{5}}{\frac{1}{5}} = 6\text{)}$$

Draw arrows on the above scales to show the operation in these problems. Write the answers. (We use the horizontal line to show division.)

$$\frac{\frac{8}{5}}{\frac{1}{5}} = (8) \qquad \frac{\frac{5}{5}}{\frac{1}{5}} = (5)$$

$$\frac{\frac{2}{5}}{\frac{1}{5}} = (2) \qquad \frac{\frac{1}{5}}{\frac{1}{5}} = (1)$$

$$\frac{\frac{11}{5}}{\frac{1}{5}} = (11) \qquad \frac{\frac{9}{5}}{\frac{1}{5}} = (9)$$

$$\frac{\frac{3}{5}}{\frac{1}{5}} = (3) \qquad \frac{\frac{4}{5}}{\frac{1}{5}} = (4)$$

$$\frac{\frac{15}{5}}{\frac{1}{5}} = (15) \qquad \frac{\frac{7}{5}}{\frac{1}{5}} = (7)$$

$$\frac{\frac{13}{5}}{\frac{1}{5}} = (13) \qquad \frac{\frac{6}{5}}{\frac{1}{5}} = (6)$$

$$\frac{\frac{16}{5}}{\frac{1}{5}} = (16) \qquad \frac{\frac{10}{5}}{\frac{1}{5}} = (10)$$

Compare these problems and their answers to those in Lesson Sixteen, Part 1. What do you discover?

Note to Teacher:

It is anticipated that the children will discover that multiplication by five is the same as division by $\frac{1}{5}$. (5 is the reciprocal of $\frac{1}{5}$ and vice versa)

Part 2

Can you give "d" another name?

$$\frac{\frac{d}{5}}{\frac{1}{5}} = 2 \qquad d = (2)$$

$$\frac{\frac{d}{5}}{\frac{1}{5}} = 1 \qquad d = (1)$$

$$\frac{\frac{d}{5}}{\frac{1}{5}} = 7 \qquad d = (7)$$

$$\frac{\frac{d}{5}}{\frac{1}{5}} = 6 \qquad d = (6)$$

$$\frac{\frac{d}{5}}{\frac{1}{5}} = 3 \qquad d = (3)$$

$$\frac{\frac{d}{5}}{\frac{1}{5}} = 4 \qquad d = (4)$$

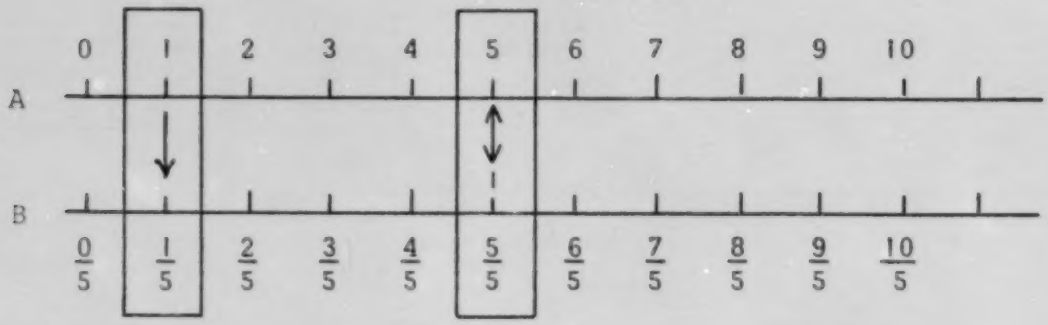
$$\frac{\frac{d}{5}}{\frac{1}{5}} = 11 \qquad d = (11)$$

$$\frac{\frac{d}{5}}{\frac{1}{5}} = 5 \qquad d = (5)$$

Lesson Eighteen

Part I

Let's look at the very same scale that helped us to divide by $\frac{1}{5}$.



To the children:

Yes, our scale also shows that $1 \longrightarrow \frac{1}{5}$. Going from a point on scale A to a matching point on scale B gives us multiplication by $\frac{1}{5}$. Therefore, going from point 4 on A scale to the matching point on B scale gives us $\frac{4}{5}$. We can write it a short way, like this: $4 \cdot \frac{1}{5} = \frac{4}{5}$

Use your number scale to discover x:

$x \cdot \frac{1}{5} = \frac{7}{5}$ $\quad x = (7)$

$x \cdot \frac{1}{5} = \frac{2}{5}$ $\quad x = (2)$

$x \cdot \frac{1}{5} = \frac{3}{5}$ $\quad x = (3)$

$x \cdot \frac{1}{5} = \frac{6}{5}$ $\quad x = (6)$

$x \cdot \frac{1}{5} = \frac{11}{5}$ $\quad x = (11)$

$x \cdot \frac{1}{5} = \frac{5}{5}$ $\quad x = (5)$

$x \cdot \frac{1}{5} = \frac{9}{5}$ $\quad x = (9)$

$x \cdot \frac{1}{5} = \frac{12}{5}$ $\quad x = (12)$

HINT: Would it help if you extended your number scale?

Part 2

Try these:

$8 \cdot \frac{1}{5} = (\frac{8}{5})$

$7 \cdot \frac{1}{5} = (\frac{7}{5})$

$5 \cdot \frac{1}{5} = (\frac{5}{5})$

$2 \cdot \frac{1}{5} = (\frac{2}{5})$

$6 \cdot \frac{1}{5} = (\frac{6}{5})$

$4 \cdot \frac{1}{5} = (\frac{4}{5})$

$10 \cdot \frac{1}{5} = (\frac{10}{5})$

$1 \cdot \frac{1}{5} = (\frac{1}{5})$

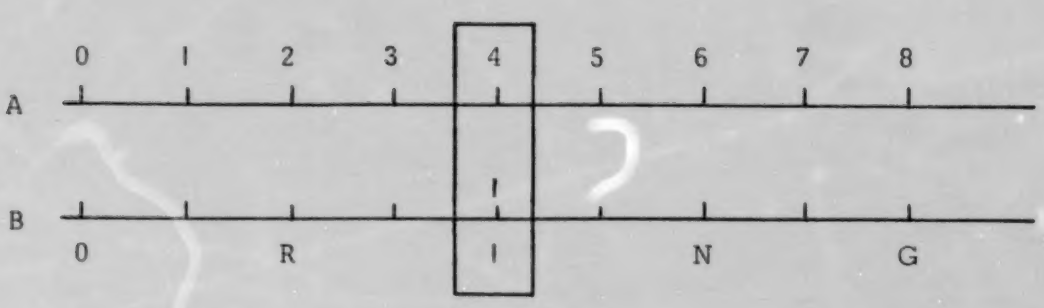
$3 \cdot \frac{1}{5} = (\frac{3}{5})$

$9 \cdot \frac{1}{5} = (\frac{9}{5})$

$11 \cdot \frac{1}{5} = (\frac{11}{5})$

$13 \cdot \frac{1}{5} = (\frac{13}{5})$

Lesson Nineteen



Review with children:

Can you name N? Yes, our key gives us the clue. $(\frac{6}{4})$

Can you explain why its name is $\frac{6}{4}$?

When we go from scale A to scale B our key tells us we are dividing by 4 ($4 \longrightarrow 1$). Therefore, 6 divided by 4 gives us the name of N.

Can you name R? $(\frac{2}{4})$ Letter "G" should be easy. $(\frac{8}{4})$

Can you solve the mystery of "y"?

$y \cdot 4 = 7$ $\quad y = (\frac{7}{4})$

$y \cdot 4 = 9$ $\quad y = (\frac{9}{4})$

$y \cdot 4 = 3$ $\quad y = (\frac{3}{4})$

$y \cdot 4 = 10$ $\quad y = (\frac{10}{4})$

$y \cdot 4 = 8$ $\quad y = (\frac{8}{4})$

$y \cdot 4 = 6$ $\quad y = (\frac{6}{4})$

$y \cdot 4 = 11$ $\quad y = (\frac{11}{4})$

$y \cdot 4 = 5$ $\quad y = (\frac{5}{4})$

Find the value of p:

$\frac{6}{4} \cdot 4 = p$

$p = (6)$

$\frac{3}{4} \cdot 4 = p$

$p = (3)$

$\frac{1}{4} \cdot 4 = p$

$p = (1)$

$\frac{4}{4} \cdot 4 = p$

$p = (4)$

$\frac{9}{4} \cdot 4 = p$

$p = (9)$

$\frac{7}{4} \cdot 4 = p$

$p = (7)$

$\frac{5}{4} \cdot 4 = p$

$p = (5)$

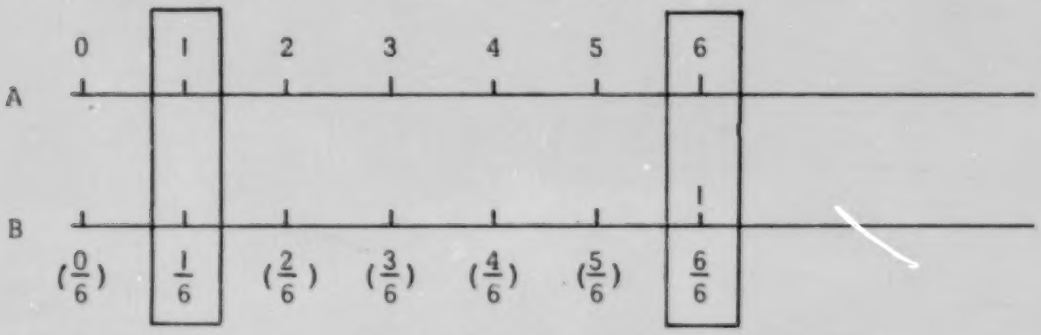
$\frac{2}{4} \cdot 4 = p$

$p = (2)$

Lesson Twenty

We have found that a number scale can help us in many ways. Look at the scale below. Name the points on B. Put a box around keys.

How many problems could we solve with this scale? Write them on your worksheet. Do not include the answers. When you have written all the possible problems, exchange worksheets with your neighbor. Use the number scale to solve your neighbor's problems.



Possible Answers: If scale is extended there are even more possibilities.

$(1 \cdot 6 = 6)$ $\quad$ $(\frac{0}{6} \cdot 6 = 0)$

$(\frac{6}{6} \cdot 6 = 6)$ $\quad$ $(1 \cdot \frac{1}{6} = \frac{1}{6})$

$(\frac{5}{6} \cdot 6 = 5)$ $\quad$ $(2 \cdot \frac{1}{6} = \frac{2}{6})$

$(\frac{4}{6} \cdot 6 = 4)$ $\quad$ $(3 \cdot \frac{1}{6} = \frac{3}{6})$

$(\frac{3}{6} \cdot 6 = 3)$ $\quad$ $(4 \cdot \frac{1}{6} = \frac{4}{6})$

$(\frac{2}{6} \cdot 6 = 2)$ $\quad$ $(5 \cdot \frac{1}{6} = \frac{5}{6})$

$(\frac{1}{6} \cdot 6 = 1)$ $\quad$ $(6 \cdot \frac{1}{6} = \frac{6}{6})$

$(5 \div 6 = \frac{5}{6})$ $(6 \div 6 = \frac{6}{6})$

$(3 \div 6 = \frac{3}{6})$ $(4 \div 6 = \frac{4}{6})$

$(1 \div 6 = \frac{1}{6})$ $(2 \div 6 = \frac{2}{6})$

$(\frac{\frac{2}{6}}{\frac{1}{6}} = 2)$ $(\frac{\frac{1}{6}}{\frac{1}{6}} = 1)$

$(\frac{\frac{4}{6}}{\frac{1}{6}} = 4)$ $(\frac{\frac{3}{6}}{\frac{1}{6}} = 3)$

$(\frac{\frac{6}{6}}{\frac{1}{5}} = 6)$ $(\frac{\frac{5}{6}}{\frac{1}{6}} = 5)$

Note to Teacher:

Give children a sufficient amount of time to solve problems. Then demonstrate procedure on chalkboard with the help of children so that they may check their answers.

Lesson Twenty-one

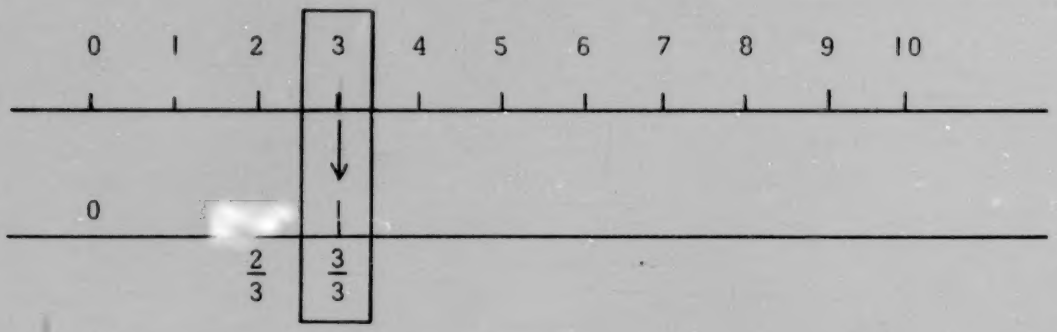
Note to teacher: This lesson may be used for enrichment. It may be too difficult for some children to construct these scales on their own.

Part I

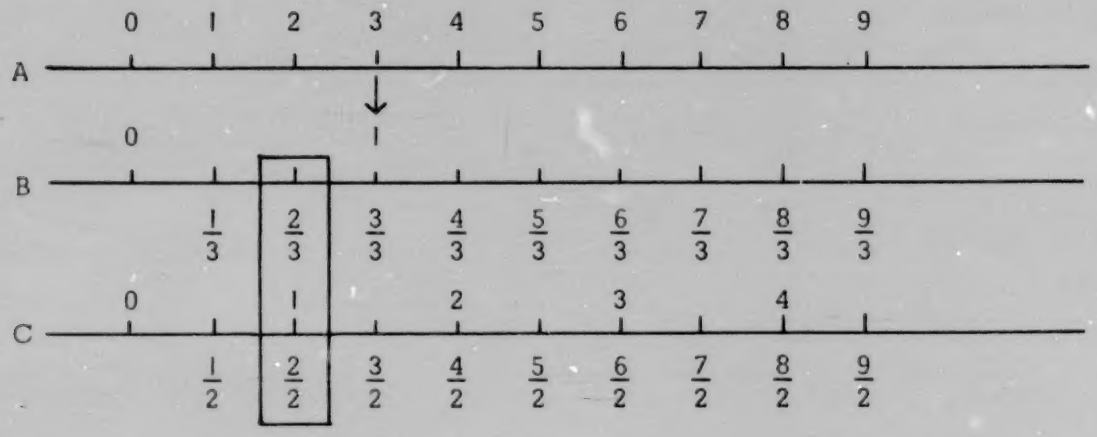
Develop the following on the chalkboard with the help of children.

Here is a problem: $\dfrac{\frac{5}{3}}{\frac{2}{3}}$

Could we construct a scale which could do this operation for us? Let's try. First, we show a scale where $\frac{2}{3} \longrightarrow 1$. This will give us division by $\frac{2}{3}$ because whatever goes to one gives us division by that number. In order to find the point $\frac{2}{3}$ we must construct a scale where $3 \longrightarrow 1$.



Now we can locate point $\frac{2}{3}$. We then construct a third number line. Point "1" on this line will match point $\frac{2}{3}$ of the second line. $\frac{2}{3} \longrightarrow 1$.



Name the other points on our last line. (The names for these points are discovered through the key $2 \longrightarrow 1$ or $A_2 \longrightarrow C_1$.)

Is it possible to solve our problem with this scale?

Locate point $\frac{5}{3}$ on B. To go from the second scale (B) to the third scale (C) gives us division by $\frac{2}{3}$ ($\frac{2}{3} \longrightarrow 1$).

Therefore $\dfrac{\frac{5}{3}}{\frac{2}{3}} = \frac{5}{2}$

Part 2

Can you divide by $\frac{2}{3}$? Construct your scale to solve these problems:

> Note to Teacher:
> Review the procedure with the children so that they may construct their own scales.

$$\frac{\frac{1}{3}}{\frac{2}{3}} = (\tfrac{1}{2}) \qquad \frac{\frac{3}{3}}{\frac{2}{3}} = (\tfrac{3}{2}) \qquad \frac{\frac{2}{3}}{\frac{2}{3}} = (1)$$

$$\frac{\frac{4}{3}}{\frac{2}{3}} = (2) \qquad \frac{\frac{6}{3}}{\frac{2}{3}} = (3) \qquad \frac{\frac{7}{3}}{\frac{2}{3}} = (\tfrac{7}{2})$$

$$\frac{\frac{8}{3}}{\frac{2}{3}} = (4) \qquad \frac{\frac{5}{3}}{\frac{2}{3}} = (\tfrac{5}{2}) \qquad \frac{\frac{9}{3}}{\frac{2}{3}} = (\tfrac{9}{2})$$

Part 3

Let's try multiplication by $\frac{2}{3}$. Can you solve these problems?

$\frac{5}{2} \cdot \frac{2}{3} = B$

$B = (\frac{5}{3})$

$\frac{2}{2} \cdot \frac{2}{3} = B$

$B = (\frac{2}{3})$

$\frac{4}{2} \cdot \frac{2}{3} = B$

$B = (\frac{4}{3})$

$\frac{1}{2} \cdot \frac{2}{3} = B$

$B = (\frac{1}{3})$

$\frac{3}{2} \cdot \frac{2}{3} = B$

$B = (1)$

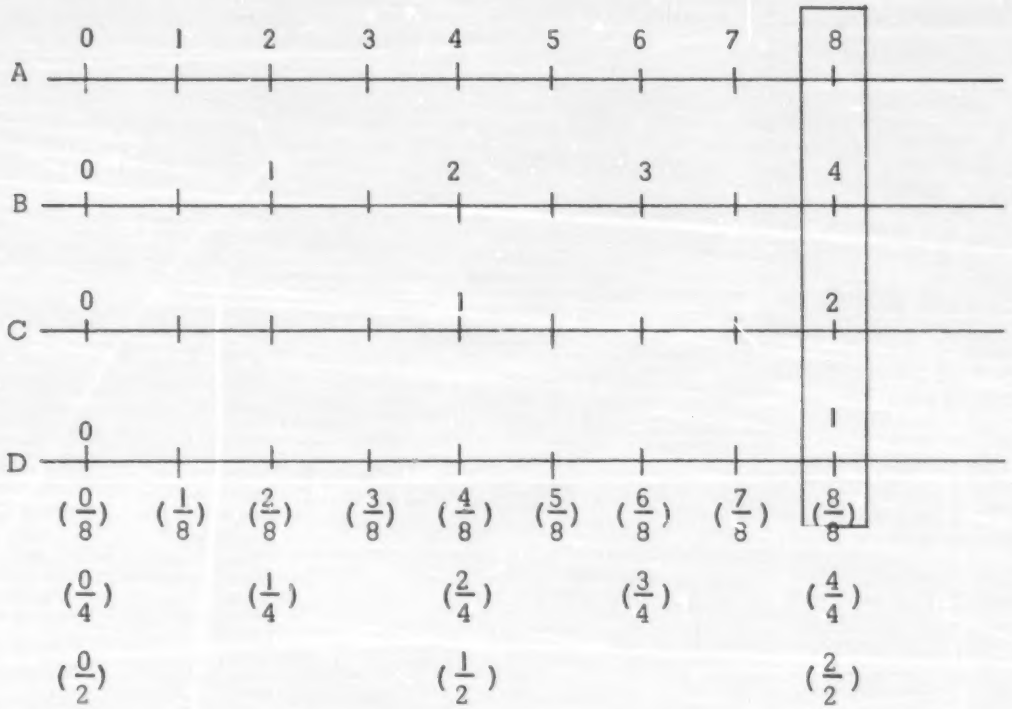
$\frac{6}{2} \cdot \frac{2}{3} = B$

$B = (2)$

Lesson Twenty-two

Part 1

Name the points on D! According to these scales, can they have more than one name?



Note to Teacher:

The children should recognize the need to name the points as division by 8 $(8 \longrightarrow 1)$ and division by 4 $(4 \longrightarrow 1)$.

Part 2

Use the above scales to find p.

$\frac{1}{4} = \frac{p}{8}$ $p = (2)$ $\frac{2}{4} = \frac{p}{8}$ $p = (4)$

$\frac{4}{4} = \frac{p}{8}$ $p = (8)$ $\frac{3}{4} = \frac{p}{8}$ $p = (6)$

$\frac{2}{8} = \frac{p}{4}$ $p = (1)$ $\frac{8}{8} = \frac{p}{4}$ $p = (4)$

$\frac{6}{8} = \frac{p}{4}$ $p = (3)$ $\frac{4}{8} = \frac{p}{4}$ $p = (2)$

Extend the scale and solve for p:

$\frac{5}{4} = \frac{p}{8}$ $p = (10)$ $\quad$ $\frac{10}{8} = \frac{p}{4}$ $p = (5)$

$\frac{12}{8} = \frac{p}{4}$ $p = (6)$ $\quad$ $\frac{3}{2} = \frac{p}{8}$ $p = (12)$

$\frac{3}{2} = \frac{p}{4}$ $p = (6)$

Can you add $\frac{1}{8} + \frac{3}{4}$? $(\frac{7}{8})$

Construct your sliding scale.

Use your scales to solve for y.

$\frac{2}{8} + \frac{2}{4} = y$ $\quad$ $\frac{3}{4} + \frac{5}{8} = y$

$y = (\frac{6}{8})$ $\quad$ $y = (\frac{11}{8})$

$\frac{1}{2} + \frac{3}{8} = y$ $\quad$ $\frac{7}{8} + \frac{2}{4} = y$

$y = (\frac{7}{8})$ $\quad$ $y = (\frac{11}{8})$

$\frac{2}{4} + \frac{1}{8} = y$ $\quad$ $\frac{9}{8} + \frac{1}{4} = y$

$y = (\frac{5}{8})$ $\quad$ $y = (\frac{11}{8})$

$\frac{3}{2} + \frac{3}{8} = y$ $\quad$ $\frac{3}{4} + \frac{1}{8} = y$

$y = (\frac{15}{8})$ $\quad$ $y = (\frac{7}{8})$

Can you find the value of x?

If $\frac{3}{x} + \frac{1}{8} = \frac{7}{8}$, then $x = (4)$

If $\frac{1}{2} + \frac{3}{x} = \frac{7}{8}$, then $x = (8)$

If $\frac{2}{4} + \frac{x}{8} = \frac{5}{8}$, then $x = (1)$

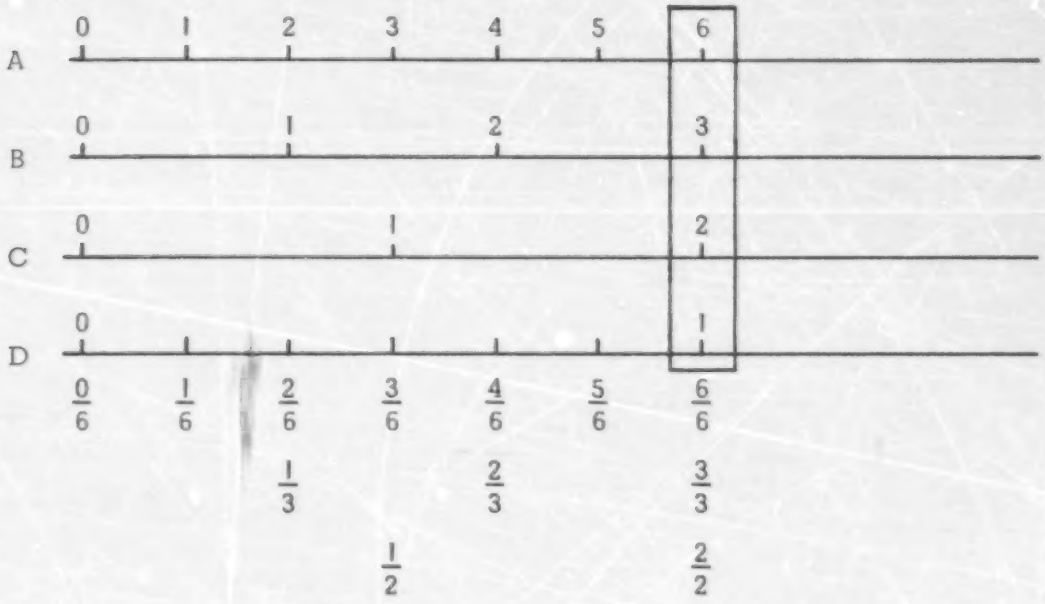
If $\frac{5}{8} + \frac{3}{x} = \frac{11}{8}$, then $x = (4)$

If $\frac{9}{8} + \frac{1}{x} = \frac{11}{8}$, then $x = (4)$

Lesson Twenty-three

Here we have a scale which consists of four number lines. Let's see how many different names we can give to the points on D.

> See Note to Teacher at the beginning of Lesson Twelve. Reinforce the idea that a fraction can have different names as $\frac{1}{2}$, $\frac{3}{6}$.



I. Key: $6 \longrightarrow 1$ (Points on line A $\longrightarrow$ Points on line D).
Name these points on D in red pencil.

II. Key: $3 \longrightarrow 1$ (Points on line B $\longrightarrow$ D).
Name the points on D in ink.

III. Key: $2 \longrightarrow 1$ (Points on line C $\longrightarrow$ D).
Name the points on D in pencil.

Part I

Refer to the scale to find n:

$\frac{3}{3} = \frac{n}{6}$ n = (6)

$\frac{1}{3} = \frac{n}{6}$ n = (2)

$\frac{1}{2} = \frac{n}{6}$ n = (3)

$\frac{2}{3} = \frac{n}{6}$ n = (4)

$\frac{2}{2} = \frac{n}{6}$ n = (6)

Extend the scale and solve for n:

$\frac{5}{3} = \frac{n}{6}$ n = (10)

$\frac{3}{2} = \frac{n}{6}$ n = (9)

$\frac{4}{3} = \frac{n}{6}$ n = (8)

$\frac{4}{2} = \frac{n}{6}$ n = (12)

Part 2

We know that it's easy to add fractions when they have the same denominators. But could we add $\frac{1}{3} + \frac{1}{2}$? (Yes.)

We need to use our sliding scale. Cut out the sliding scale from your worksheet. Locate point $\frac{1}{3}$ on your first scale. Place zero of your sliding scale at that point. Locate point $\frac{1}{2}$. We read our answer on first scale $\frac{5}{6}$.

Now add these fractions. Extend the scale, if necessary.

$\frac{2}{3} + \frac{3}{2} = (\frac{13}{6})$

$\frac{1}{6} + \frac{2}{2} = (\frac{7}{6})$

$\frac{5}{6} + \frac{1}{3} = (\frac{7}{6})$

$\frac{3}{3} + \frac{5}{6} = (\frac{11}{6})$

$\frac{1}{2} + \frac{2}{6} = (\frac{5}{6})$

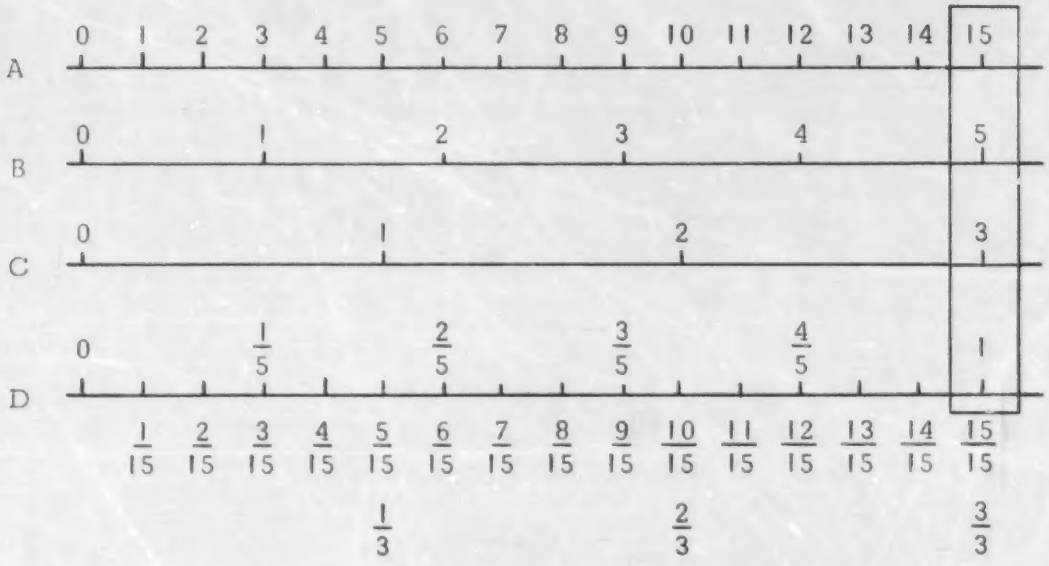
$\frac{1}{3} + \frac{3}{6} = (\frac{5}{6})$

$\frac{4}{3} + \frac{7}{6} = (\frac{15}{6})$

$\frac{1}{2} + \frac{1}{3} = (\frac{5}{6})$

Lesson Twenty-four

Can this scale help us to add $\frac{1}{5} + \frac{1}{3}$?



First, locate $\frac{1}{5}$ on D scale. What is another name for $\frac{1}{5}$? ($\frac{3}{15}$). Now locate $\frac{1}{3}$ on sliding scale D. What other name is there for $\frac{1}{3}$? ($\frac{5}{15}$). Can we add $\frac{1}{5} + \frac{1}{3}$? (Yes.) What made it easy for us to find the answer? We have found the other names that enable us to do it. Our answer is ($\frac{8}{15}$).

Solve these problems. Rewrite the sentences using the names that will enable us to find the answers on our scale.

$$\frac{2}{5} + \frac{1}{15} = (\frac{6}{15} + \frac{1}{15} = \frac{7}{15})$$

$$\frac{4}{5} + \frac{1}{3} = (\frac{12}{15} + \frac{5}{15} = \frac{17}{15})$$

$$\frac{2}{3} + \frac{1}{15} = (\frac{10}{15} + \frac{1}{15} = \frac{11}{15})$$

$\frac{1}{5} + \frac{2}{3} = (\frac{3}{15} + \frac{10}{15} = \frac{13}{15})$

$\frac{2}{3} + \frac{2}{5} = (\frac{10}{15} + \frac{6}{15} = \frac{16}{15})$

$\frac{4}{5} + \frac{2}{3} = (\frac{12}{15} + \frac{10}{15} = \frac{22}{15})$

$\frac{1}{15} + \frac{1}{3} = (\frac{1}{15} + \frac{5}{15} = \frac{6}{15})$

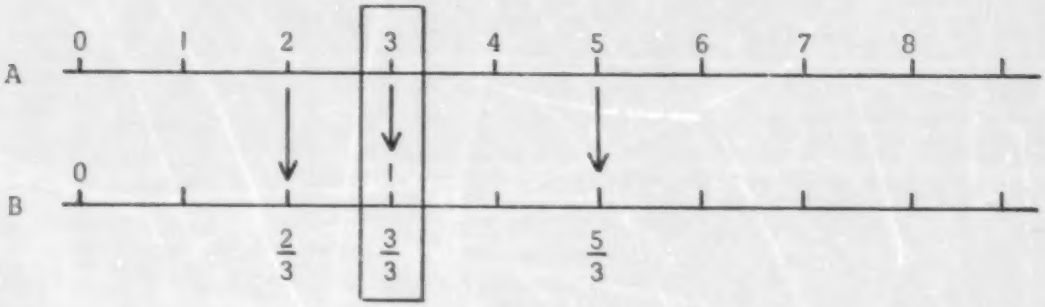
$\frac{3}{5} + \frac{3}{15} = (\frac{9}{15} + \frac{3}{15} = \frac{12}{15})$

$\frac{1}{5} + \frac{4}{15} = (\frac{3}{15} + \frac{4}{15} = \frac{7}{15})$

$\frac{2}{5} + \frac{2}{3} = (\frac{6}{15} + \frac{10}{15} = \frac{16}{15})$

Lesson Twenty-five

Which number is bigger? $\frac{2}{3}$ or $\frac{5}{3}$



Locate the point $\frac{2}{3}$ on B.

Locate the point $\frac{5}{3}$ on B.

On scale A, point 5 is to the right of 2.

On scale B, point $\frac{5}{3}$ is directly below point 5 and point $\frac{2}{3}$ is directly below 2.

Therefore $\frac{5}{3}$ is to the right of $\frac{2}{3}$.

Therefore $\frac{5}{3}$ is bigger than $\frac{2}{3}$.

Note to Teacher:

In the following problems the children should go through the same kind of reasoning in order to determine which is the bigger fraction. To answer those questions which involve comparisons of fractions with different denominators, it is useful to refer to the section in equivalent fractions (Lessons 22-24). For example, to compare $\frac{2}{3}$ and $\frac{5}{6}$ we find that $\frac{2}{3}$ is the same as $\frac{4}{6}$. Point $\frac{5}{6}$ is to the right of point $\frac{4}{6}$ on the scale. Therefore $\frac{5}{6} > \frac{4}{6}$.

Which is bigger? Rewrite fractions using symbols $>$, $<$

$\frac{1}{3}$ or $\frac{1}{6}$

$(\frac{1}{3} > \frac{1}{6})$

$\frac{3}{4}$ or $\frac{5}{8}$

$(\frac{3}{4} > \frac{5}{8})$

$\frac{1}{4}$ or $\frac{1}{8}$

$(\frac{1}{4} > \frac{1}{8})$

$\frac{2}{4}$ or $\frac{5}{8}$

$(\frac{2}{4} < \frac{5}{8})$

$\frac{3}{4}$ or $\frac{3}{8}$

$(\frac{3}{4} > \frac{3}{8})$

$\frac{7}{15}$ or $\frac{2}{5}$

$(\frac{7}{15} > \frac{2}{5})$

$\frac{1}{5}$ or $\frac{5}{15}$

$(\frac{1}{5} < \frac{5}{15})$

$\frac{9}{15}$ or $\frac{4}{5}$

$(\frac{9}{15} < \frac{4}{5})$

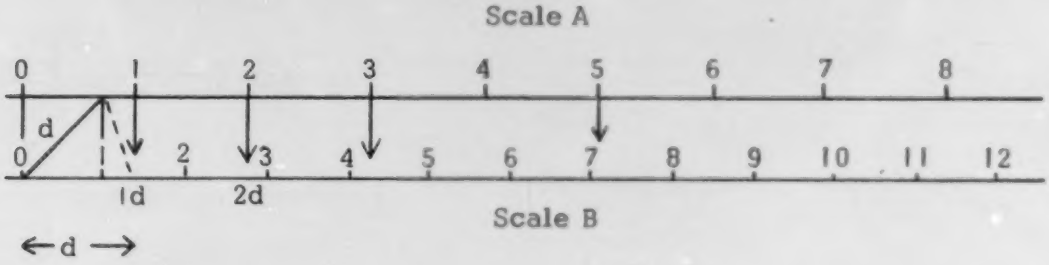
$\frac{2}{3}$ or $\frac{3}{5}$

$(\frac{2}{3} > \frac{3}{5})$

Lesson Twenty-six

Note to Teacher:

We have learned that there are numbers that are not rational. In the following scale the length of d is evidence of the existence of such numbers. We can only approximate their value. This is done by locating the irrational number as a point between two known integers. We call this "sandwiching." We use the symbols > to represent "greater than" and < to represent "less than." Review these symbols and their relationships with the children before you proceed. For example: $1 < \frac{5}{4} < 2$.



Point "1" on scale A goes to point "d" on scale B. Therefore $1 \longrightarrow d$ or we can say that going from a point on scale A to a point directly below on scale B is multiplication by d.

If we wish to locate point 2d on scale B we can say that $2 < 2d < 3$. (2d is sandwiched between 2 and 3.)

Locate the following on scale B:

Scale A	Scale B
6d	$(8 < 6d < 9)$
4d	$(5 < 4d < 6)$
7d	$(9 < 7d < 10)$
3d	$(4 < 3d < 5)$
5d	$(7 < 5d < 8)$
8d	$(11 < 8d < 12)$
d	$(1 < d < 2)$
2d	$(2 < 2d < 3)$